# Sound radiation of the end of cylindrical duct application on industrial stacks

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In order to determine the sound field emitted in the environment by the openings of industrial stacks, a study relating to the acoustic radiation of the semi-infinite cylindrical ducts opened at their end was carried out. First, a study of the principal publications available on the subject was undertaken. This made it possible to establish a calculation model of the directivity function of a stack opening. This model is based on the theory of H. Levine and J. Schwinger for the fundamental mode, and on a publication of G.F. Homicz and J.A. Lordi for higher order modes. In order to validate the model, measurements on a scale model were done in an anechoic room. A good correlation between theory and experimental results appear, especially at low frequencies, where the number of radiating modes is low. Furthermore, measurements on a real industrial site have been done, showing a satisfying correlation between theory and experimentation.

# 1 Introduction

In the scope of impact studies of industrial facilities on their environment, the acoustic engineer has to estimate in any point noise levels radiated by stack openings. In order to estimate the power radiated by these sources from a noise level measurement, the radiation directivity of an unflanged open duct must be known. This article presents a recap of the literature on the subject as well as the experimental validation of the corresponding theories.

# 2 Theoretical considerations

## 2.1 Fundamental mode

With regard to the fundamental mode, the theory was initially established per H. Levine and J. Schwinger in 1947 [1]. The mouth of the cylindrical opening is considered as an unflanged rigid circular plane piston. All elementary surfaces of the opening are supposed to vibrate in phase ("piston" mode).

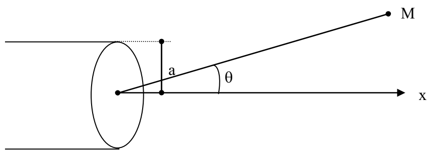


Figure 1. Unflanged circular plane piston.

With these assumptions, and with the notations of figure 1, the expression of the directivity function is as follows:

$$H(\theta)=\frac{4}{\pi\sin^2\theta}\frac{J_1(k_0 a\sin\theta)}{\left[(J_1(k_0a\sin\theta))^2+(N_1(k_0a\sin\theta))^2\right]^{\frac{1}{2}}}\frac{|R|}{1-|R|^2}$$

$$\times\exp\left\{\frac{2k_0a\cos\theta}{\pi}P\int_0^{k_0a}\frac{x\tan^{-1}\left(\frac{-J_1(x)}{N_1(x)}\right)}{\left[x^2-(k_0a\sin\theta)^2\right]\left[x^2+(k_0a)^2\right]^{\frac{1}{2}}}dx\right\} \quad (1)$$

Where :

$k_0$ wave number ($m^{-1}$),

$\omega$ pulsation ($s^{-1}$),

c speed of sound ($ms^{-1}$),

$J_1$ 1st order Bessel function,

$N_1$ 1st order Neumann function,

P signifies that the integral is to be understood as a Cauchy principal value,

R reflection coefficient of the velocity potential. Its expression is :

$$|R|=\exp\left\{-\frac{2k_0a}{\pi}\int_0^{ka}\frac{\tan^{-1}\left(-J_1(x)/N_1(x)\right)}{x\left[(k_0a)^2-x^2\right]^{\frac{1}{2}}}dx\right\} \quad (2)$$

## 2.2 Higher order modes

For the higher order modes, several authors propose an exact solution. We will quote G.F. Homicz and J.A. Lordi [2] who propose a demonstration based on the technique of Wiener-Hopf. The power radiated by the mode $A_{m,s}$ per solid unit of angle is written:

$$P(\theta)=\frac{P_i k_0 k_x^s}{\pi^2\left(k_0\cos\theta-k_x^s\right)}\frac{J'_m(k_0a\sin\theta)}{\left|H'_m(k_0a\sin\theta)\right|}\prod_{\substack{n=1\\n\neq s}}^{N}\frac{k_x^n+k_x^s}{k_x^n-k_x^s}\prod_{n=1}^{N}\frac{k_x^n-k_0\cos\theta}{k_x^n+k_0\cos\theta}$$

$$\times\exp\left\{\Re\left[S(k_x^s a)-S(k_0a\cos\theta)\right]\right\} \quad (3)$$

Where :

$P_i$ incident power of the mode (W),

$k_0$ wave number ($m^{-1}$),

$\omega$ pulsation ($s^{-1}$),

c speed of sound ($ms^{-1}$),

$k_x$ axial wave number ($m^{-1}$),

$k_r$ radial wave number($m^{-1}$),

$J_m$ Bessel function of the first kind of order m,

$H_m$ Hankel fonction of the first kind of order m.

Function $\Re[S]$ is given by:

$$\Re[S(\zeta)] = \frac{1}{\pi} P \int_{-ka}^{+ka} \frac{\Omega(\kappa a)}{k-\zeta} dk \quad \text{with} \quad \kappa = \sqrt{k_0^2 - k^2} \qquad (4)$$

$$\Omega(k) = \tan^{-1} \frac{Y'_m(k)}{J'_m(k)} \pm \frac{\pi}{2} \quad \begin{cases} + \text{ if } m = 0 \\ - \text{ if } m \neq 0 \end{cases} \qquad (5)$$

And:

$\Omega(0) = 0$
$\Omega(j'_{ms}) = (s-1)\pi$ si $m \neq 0$
$\Omega(j'_{ms}) = s.\pi$ si $m = 0$

P signifies that the integral is to be understood as a Cauchy principal value.

$j'_{ms}$ are the roots of the function $J'_m$, introduced to take into account Neumann conditions on the internal duct wall.

Moreover, k, $k_x$ et $k_r$ are related by the dispersion relation:

$$k_x^2 = k_0^2 - k_r^2 \qquad (6)$$

With $k_0 = \frac{\omega}{c}$ et $k_r = \frac{j'_{ms}}{a}$

This relation is fundamental because, for a fixed value of $k_0a$, it determines if the mode is propagative, i.e. when $k_x$ is real, or evanescent, when $k_x$ is imaginary.

It rises from the expression (3) that the number of radiant modes increases with the value of $k_0a$, therefore with the diameter and the frequency.

Moreover, S.T. HOCTER proposes in 2000 an approximation with a simpler analytical expression [5], based on the geometrical theory of diffraction (see reference [3] for details). It is this formulation which was used to compare the theoretical and experimental results.

# 3 Experimental results

## 3.1 In anechoic room

In order to validate the theory, tests were carried out on a scale model in an anechoic room. A loudspeaker generating a pink noise was positioned in an insulated box. The sound pressure level was measured per octave band on a 0.3 m radius circle, centered on the opening of a 1 m long and 0.05 m diameter duct, with an angular step of 15° (cf. . 2).

Figure 3 presents the theoretical and experimental values of the directivity function10logH(θ), in dB, for octave bands 125 Hz, 250 Hz, 1000 Hz, 2000 Hz, 4000 Hz et 8000 Hz.

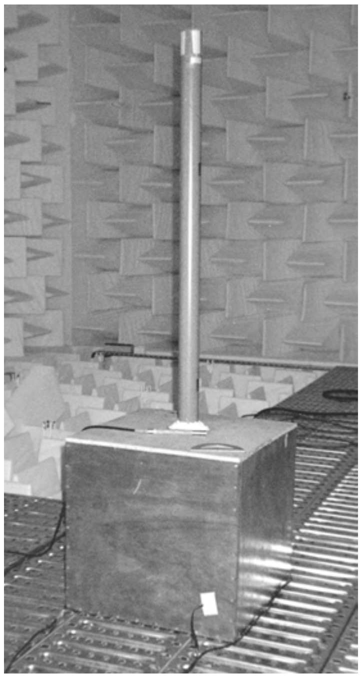

Figure 2. Scale model for laboratory measurement.

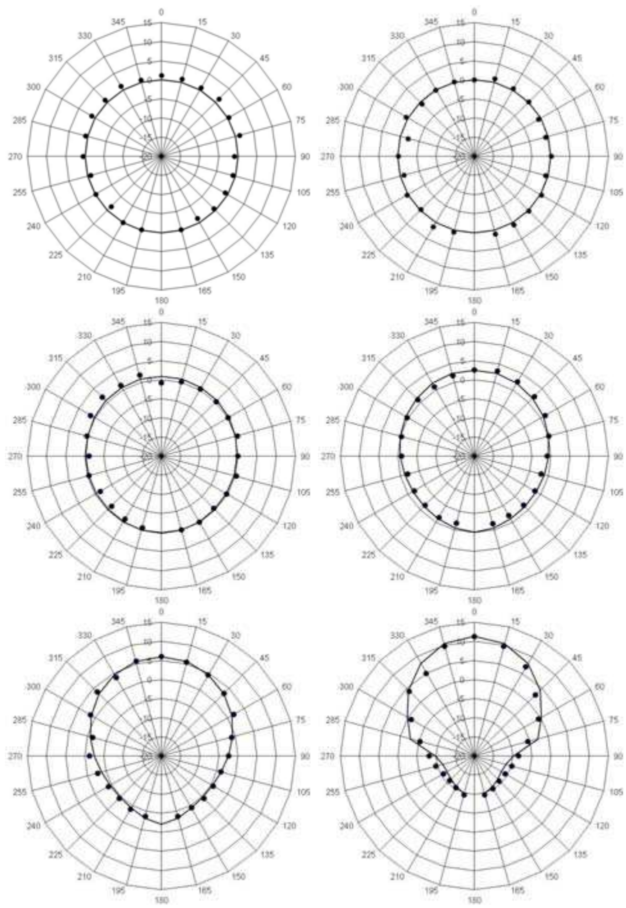

Figure 3. Angular directivity patterns, in dB, for octave bands 125 Hz, 250 Hz, 1000 Hz, 2000 Hz, 4000 Hz and 8000 Hz. Points correspond to experimental values, curves in full features correspond to computed values.

### 3.2 In situ

For an industrial impact study, the sound pressure level radiated by a real stack with a diameter of 2 m was measured in two distinct points. On the basis of one of two measurements, the noise level at the second point was estimated (cf. figure 4).

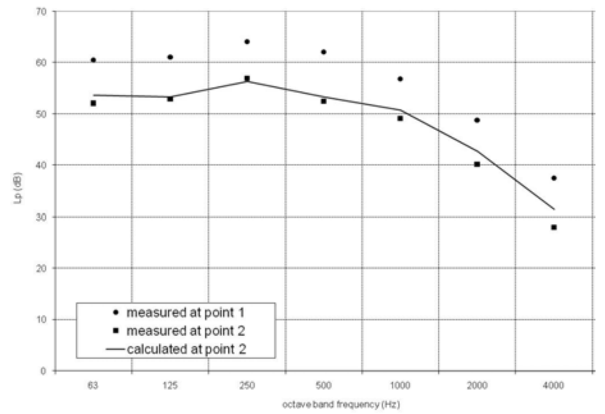


Figure 4. Sound pressure level measured and calculated at a point (point 2) starting from a value measured in another point (point 1).

Results are satisfying, in particular at low frequencies, for which the number of radiant modes is small. Result dispersion is observed when frequency, and the amount of radiant modes increase.

## 4 Discussion

With regard to octave bands 125 Hz, 250 Hz, 1 kHz, 2 kHz and 4 kHz, for which only piston mode radiates, one calculates the difference between experimental and theoretical values varying from -1.8 dB to +1.3 dB. For the 8000 Hz octave band, the difference varies from -3 dB to +1.9 dB. The agreement between measurement and theory is thus degraded in the case of multimodal radiation. This may be explained by the assumption of equi-contribution of radiant modes. Results are however satisfying.

In the case of the 8000 Hz radiation, it is interesting to visualize directivity pattern of the piston mode, and to compare it with the measured values (cf. figure 5). This curve shows that the fundamental mode is the only one that radiates in the axis of the opening (cancellation of the Bessel function of higher order than 0 in the axis). In the case of the model, this mode contributes even to most of the energy radiated in the superior half space. On the other hand, the theoretical values for the inferior half space, under the plan of the opening, are lower than the experimental values. The contribution of higher order other modes is thus not negligible for these points.

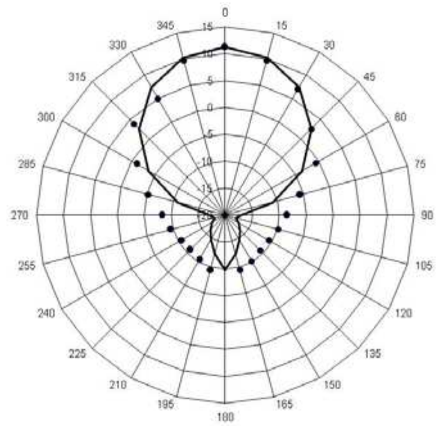

Figure 5. Angular directivity pattern, in dB, in octave band 8000 Hz. Points correspond to the experimental values, the full curve corresponds to calculation of directivity function of the fundamental mode.

In the case of a real industrial stack, annoyance is perceived in the lower half space in most cases. Unfortunately, as it was previously specified, information concerning modal distribution of sound sources at the base of the smokestack are only very seldom available. The assumption of equi-contribution of radiation modes is thus necessary in the most cases.

However, this assumption must be used with caution. Indeed, in the case of the studied model, with a 5 cm diameter duct, only five modes are to be taken into account for the evaluation of directivity. On the other hand, in the case of a real smokestack, a great number of modes are likely to radiate. For example, in the case of a 1 meter diameter stack, 91 modes are supposed to radiate in octave band 2000 Hz. Thus, the equi-contribution assumption becomes risky in this case.

## 5 Conclusion

This study permitted to establish a calculation model of the directivity function of a cylindrical stack opening.

It however showed a reduction in the correlation between measurement and theory when frequency and diameter, and thus the number of radiant modes, increase. This phenomenon is related to the assumption of an energetical equi-contribution of radiant modes. One must bear in mind that the adopted model allows to evaluate the pace of the function of total directivity, but great caution is to be adopted for the exploitation of numerical values when a great number of modes must be taken into account. This model can thus be considered as reliable for low frequencies. This point is important because sound annoyance generated by smokestacks is generally noted at low frequencies.

Moreover, it will be noted that in this study, the movement of the fluid in the duct is not taken into account. Indeed, in most industrial cases, the Mach number rarely exceeds 0,01. The contribution of flow can thus be generally regarded as negligible.

In addition, gases in industrial ducts can reach high temperatures. Consequently, temperature gradients between the plan of the opening and the point of reception are foreseeable. Propagation conditions of the signal are thus likely to be modified by this phenomenon.

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