If a sample of absorptive material having an area S_1 is placed on the floor and the test repeated, the new reverberation time is

$$T_{60}\,(2) = \frac{0.161\ V}{S_T\,\overline{\alpha} - S_1\,\alpha_0 + S_1\,\alpha_1} \tag{8.76}$$

where α_0 is the absorption coefficient of the covered portion of the floor and α_1 is the absorption coefficient of the sample material under test. Combining Eqs. 8.75 and 8.76 we obtain the desired coefficient

$$\alpha_1 = \alpha_0 + \frac{0.161\ V}{S_1}\left(\frac{1}{T_{60}(2)} - \frac{1}{T_{60}(1)}\right) \tag{8.77}$$

In these tests there is some dependence on the position of the sample in the room. Materials placed in the center of a surface are more effective absorbers, and yield higher absorption coefficients, than materials located in the corners. This is because the average particle velocity is higher there. There are also diffraction effects and edge absorption attributable to the sides of the sample. For these reasons Sabine absorption coefficients that are greater than one sometimes are obtained and must be used with caution in the Norris Eyring equation.

8.5 REVERBERANT FIELD EFFECTS

Energy Density and Intensity

We have seen that, as the modal spacing gets closer and closer together, it becomes less useful to consider individual modes and we must seek other ways of describing the behavior of sound in a room. One concept is the energy density . A plane wave moves a distance c_0 in one second and carries an energy per unit area equal to its intensity, I. The direct-field energy density D_d per unit volume is

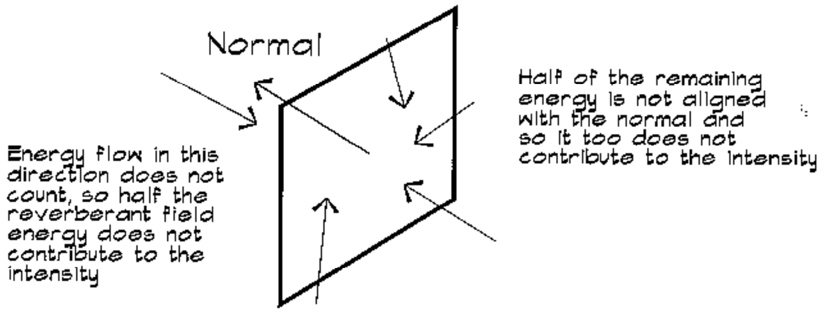
$$D_d = \frac{I_d}{c_0} = \frac{p^2}{\rho_0\, c_0^2} \tag{8.78}$$

where p^2 is the rms acoustic pressure.

The energy density in a diffuse field has the same relationship to the pressure squared, which is not a vector quantity, but a different relationship to the intensity. In a diffuse field the sound energy can be coming from any direction. The intensity is defined as the power passing through an area in a given direction. In a diffuse field, half the energy is passing through the area plane in the opposite direction to the one of interest. When we integrate the energy incident on the area in the remaining half sphere, the cosine term reduces the intensity by another factor of two. Thus in a reverberant field the intensity is only a quarter of the total power passing through the area. This is shown in Fig. 8.12.

$$I_r = \frac{1}{4}\left(\frac{p^2}{\rho_0\, c_0}\right) \tag{8.79}$$

FIGURE 8.12 Intensity in a Reverberant Field



Semireverberant Fields

Occasionally we encounter a semireverberant field, where energy falls onto one side of a plane with equal probability from any direction. Most often this occurs when sound is propagating from a reverberant field through an opening in a surface of the room. Under these conditions the power passing through the plane of an opening having area S_w is given by

$$W_{sr} = \frac{S_w}{2}\left(\frac{p^2}{\rho_0 c_0}\right) \tag{8.80}$$

Room Effect

When a sound source that emits a sound power W_S is placed in a room, the energy density will rise until the energy flow is balanced between the energy being created by the source and the energy removed from the room due to absorption. After a long time the total energy in a room having a volume V due to a source having a sound power W_S is

$$V D_r = \frac{W_S \Lambda}{c_0}\left[1 + (1-\overline{\alpha}) + (1-\overline{\alpha})^2 + \cdots\right] = \frac{W_S \Lambda}{c_0 \overline{\alpha}} \tag{8.81}$$

which has been simplified using the limit of a power series for $\overline{\alpha}^2 < 1$

$$V D_r = \frac{4 W_S V}{c_0 S_T \overline{\alpha}} \tag{8.82}$$

and the sound pressure in the room will be

$$\frac{p^2}{\rho_0 c_0} = \frac{4 W_S}{S_T \overline{\alpha}} = \frac{4 W_S}{R} \tag{8.83}$$

Equation 8.83 is the reverberant-field contribution to the sound pressure measured in a room and can be combined with the direct-field contribution to obtain

$$\frac{p^2}{\rho_0 c_0} = \frac{Q W_S}{4 \pi r^2} + \frac{4 W_S}{R} \tag{8.84}$$

Taking the logarithm of each side we can express this equation as a level

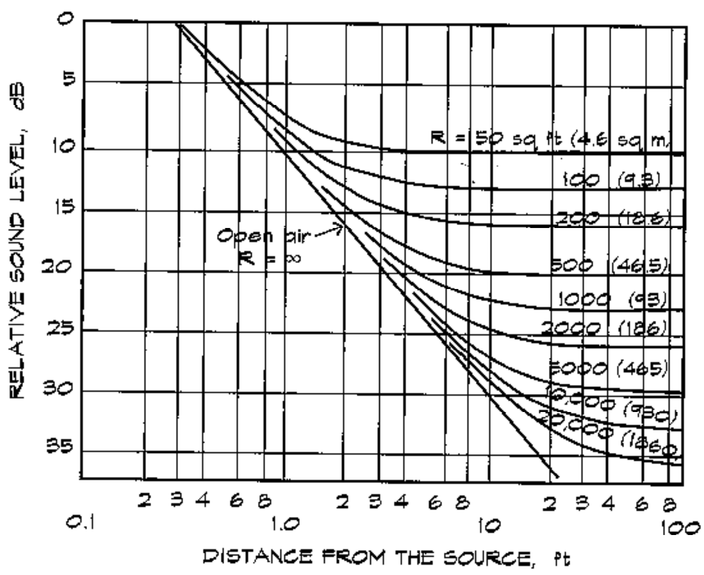
$$L_p = L_W + 10 \log \left[\frac{Q}{4 \pi r^2} + \frac{4}{R} \right] + K \tag{8.85}$$

where K is 0.1 for metric units and 10.5 for FP units. The numerical constants follow from the reasoning given in Eq. 2.67.

Equation 8.85 is based on Sabine's theory and was published in 1948 by Hopkins and Stryker. It is a useful workhorse for the calculation of the sound level in a room given the sound power level of one or more sources. It holds reasonably well where the diffuse field condition exists; that is, in relatively large rooms with adequate diffusion if we are not too close (usually within $\frac{\lambda}{2}$) to reflecting surfaces. The increase in sound pressure level due to the reverberant field over that which we would expect from free field falloff is called the room effect.

Figure 8.13 gives the result from Eq. 8.85 for various values of the room constant. Near the source the direct-field contribution is larger than the reverberant-field contribution and the falloff behavior is that of a point source in a free field. In the far field the direct-field contribution has dropped below the reverberant-field energy, and the sound pressure level is constant throughout the space. The level in the reverberant field can be reduced only by adding more absorption to the room. According to this theory, only the total amount of absorption is important, not where it is placed in the room. In practice absorption placed where the particle velocity is the highest has the greatest effect. Thus absorption mounted in a corner, where the pressure has a maximum and the velocity a minimum, would be less effective than absorption placed in the middle of a wall or other surface. Absorption, which is hung in the center of a space, has the greatest effect but this is not a practical location.

FIGURE 8.13 Difference between Sound Power and Pressure Level in a Diffuse Room Due to an Omnidirectional Source



At a given distance, known as the *critical distance*, the direct-field level equals the reverberant-field level. We can solve for the distance by setting the direct and reverberant contributions equal.

$$r_c = \sqrt{\frac{Q\,R}{16\,\pi}} \tag{8.86}$$

Beyond the critical distance the reverberant field predominates.

Radiation from Large Sources

When the source of sound is physically large, such as the wall of a room, it can radiate energy over its entire surface area. The idea of a displaced center was introduced in Eq. 2.91 to relate the sound power to the sound pressure level in free space for a receiver located close to a large radiating surface. Similarly in a reverberant space the direct and reverberant contributions are combined

$$L_p = L_W + 10\log\left[\frac{Q}{4\,\pi\left[z + \sqrt{\frac{S\,Q}{4\,\pi}}\right]^2} + \frac{4}{R}\right] + K \tag{8.87}$$

where K is 0.5 for metric and 10.5 for FP units.

As the distance z, between the surface of the source and the receiver, is reduced to zero, Eq. 8.87 can be simplified to

$$L_p \cong L_W + 10\log\left[\frac{1}{S} + \frac{4}{R}\right] + K \tag{8.88}$$

where S is the surface area of the source. When the receiver is far from the source the area contribution is small and the distance to the surface of the source and to its acoustic center are nearly equal ($z \cong r$). The equation then reverts to its previous form

$$L_p = L_W + 10\log\left[\frac{Q}{4\,\pi\,r^2} + \frac{4}{R}\right] + K \tag{8.89}$$

Departure from Diffuse Field Behavior

In the power-pressure conversion, when we do not measure the sound pressure level close to the reflecting surfaces, we neglect some energy near the boundary given in Eq. 8.74. Waterhouse (1955) has investigated this energy and has suggested the addition of a correction term to the room constant, which is only significant at low frequencies.

$$R = A\left(1 + \frac{S_T\,\lambda}{8\,V}\right) + 4\,m\,V \tag{8.90}$$

where S_T is the total surface area and V the volume of the room. The correction is used in certain test procedures (e.g., ISO 3741 and ASTM E336).

FIGURE 8.14 **Measured (Power – Pressure) Level Differences (Davis and Davis, 1978)**

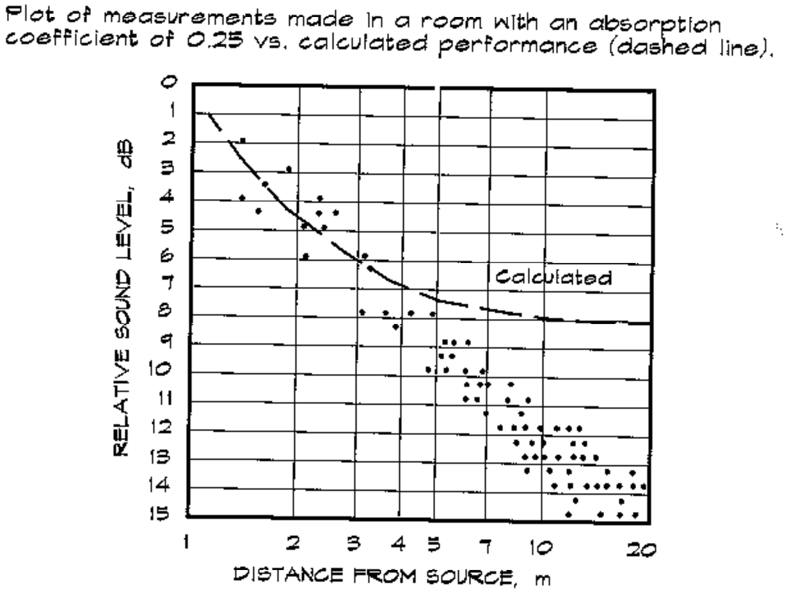
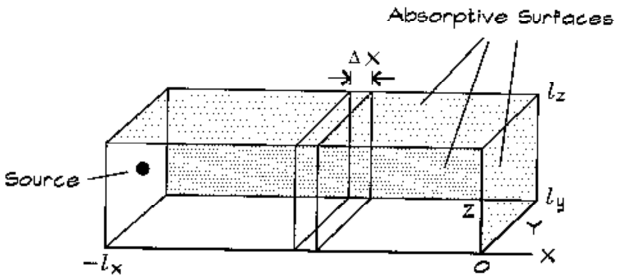


FIGURE 8.15 **Rectangular Room with Source Wall ($x = -\ell_x$), Absorbing End-Wall ($x = 0$), and Absorbing Side Surfaces ($y = \ell_y$ and $z = \ell_z$) (Franzoni, 2001)**



When rooms have a significant dimensional variation in different directions, particularly where there are low ceilings with a large amount of absorption, there is a departure from the behavior predicted by the Hopkins Stryker equation. Figure 8.14 shows measurements taken by Ogawa (1965) in Japan. A number of authors have attempted to account for this behavior by adding additional empirical terms or multipliers to the equation. Hodgson (1998) has published a review of several of these methods.

Franzoni and Labrozzi (1999) developed an empirical formula that applies to long, narrow, rectangular rooms, when the absorption is not uniformly distributed on all surfaces. For a source positioned near one wall and the geometry shown in Fig. 8.15,

$$\overline{p}^2_{rev} = \frac{4\,\rho_0\,c_0\,W}{A}\left[\frac{(1-\overline{\alpha}_{total})(1-\overline{\alpha}_{total}/2)}{(1-\overline{\alpha}_w\,\overline{S}/2)}\right] e^{-(1/2)\,\overline{\alpha}_w\,\overline{S}\overline{x}} \tag{8.91}$$

where $\overline{p}^2_{rev}$ = cross-sectionally averaged mean square acoustic pressure (Pa^2) at a *distance x from the origin (not the source)*

A = *total area of absorption in the room (sabins)*

$= S_1\,\alpha_1 + S_2\,\alpha_2 + S_3\,\alpha_3 + \cdots + S_n\,\alpha_n$

$\overline{\alpha}_{total} = A\,/\,S_{total}$

$\overline{\alpha}_w$ = total absorption of the side surfaces divided by the area of the side surfaces

$\overline{x} = x\,/\,l_x$

$\overline{S}$ = *ratio of the side wall surface area to the cross sectional surface area*

Reverberant Falloff in Long Narrow Rooms

Franzoni (2001) also published a theoretical treatment of the long-narrow room problem by considering an energy balance for diffuse-field components traveling to the right and to the left using the geometry in Fig. 8.15. She assumes that there is a locally diffuse condition, where energy incidence is equally probable in all directions from a hemisphere at a planar slice across the room, but the rightward energy does not necessarily equal the leftward energy. The total energy at a point is taken to be uncorrelated and can be expressed as the sum of the two directional components

$$\overline{p}^{\,2} = \overline{p}^{\,2}_{+x} + \overline{p}^{\,2}_{-x} \tag{8.92}$$

At a given slice the reverberant intensity, due to rightward moving waves, is

$$I_{+x} = \frac{\overline{p}^{\,2}_{+x}}{\rho_0\,c_0} \tag{8.93}$$

and similarly for the leftward moving waves.

To evaluate the effect of reflections from the side surfaces we write the mean square pressure near the wall as the sum of the incident and reflected components interacting with the sides

$$\overline{p}^{\,2}_{+x} = \overline{p}^{\,2}_{+x_{incident}} + \overline{p}^{\,2}_{+x_{incident}}(1-\alpha_w) = (2-\alpha_w)\overline{p}^{\,2}_{+x_{incident}} \tag{8.94}$$

The incident intensity into the side wall boundary (y or z) is

$$I_{sidewall} = I_s = I_y = I_z = \frac{\overline{p}^{\,2}_{+x_{incident}}}{2\,\rho_0\,c_0} = \frac{\overline{p}^{\,2}_{+x}}{2\,\rho_0\,c_0}\left[\frac{1}{2-\alpha_w}\right] \tag{8.95}$$

where $\overline{p}^{\,2}_{+x}$ = mean square pressure associated with rightward traveling waves, incident plus reflected.

If we define β as the fraction of the surface area at a cross section, covered with an absorbing material having a random incidence absorption coefficient α_w, and l_p and S as the perimeter and area of the cross section, we can write a power balance relation equating the power in to the power out of the cross section.

$$I_x\,S = \left(I_x + \frac{d\,I_x}{d\,x}\,\Delta\,x\right)S + \alpha_w\,\beta\,l_p\,\Delta\,x\,I_s \tag{8.96}$$

This can be written as a differential equation

$$\frac{d\left(\overline{p}^{\,2}_{+x}\right)}{d\,x} + \frac{\alpha_w\,\beta\,l_p}{(2-\alpha_w)\,S}\,\overline{p}^{\,2}_{+x} = 0 \tag{8.97}$$

which has a solution for right-running waves

$$\overline{p}^{\,2}_{+x} = P_{+x}\,e^{-(\alpha_w\,\beta\,l_p)/((2-\alpha_w)\,S)\,x} \tag{8.98}$$

and another for left-running waves

$$\overline{p}^{\,2}_{-x} = P_{-x}\,e^{+(\alpha_w\,\beta\,l_p)/((2-\alpha_w)\,S)\,x} \tag{8.99}$$

where P_{-x} and P_{+x} are coefficients to be determined by the boundary conditions at each end. At the absorbing end ($x = 0$) the right and left intensities are related

$$I_{-x}(0) = (1-\alpha_b)\,I_{+x}(0) \tag{8.100}$$

with α_b being the end wall random incidence absorption coefficient. The coefficients in Eqs. 8.98 and 8.99 are related

$$P_{-x} = (1-\alpha_b)\,P_{+x} \tag{8.101}$$

At the source-end wall, the power of the sources is equal to the power difference in right and left traveling waves

$$W = S\left[I_{+x}(-l_x) - I_{-x}(-l_x)\right] \tag{8.102}$$

Plugging in the mean square pressure terms and using Eq. 8.101 (Franzoni, 2001),

$$\overline{p}^{\,2}(x) = \frac{2\,\rho_0\,c_0\,W}{S}\left[\frac{\cosh(\gamma x) - \frac{1}{2}\,\alpha_b\,e^{+\gamma x}}{\sinh(\gamma l_x) + \frac{1}{2}\,\alpha_b\,e^{-\gamma l_x}}\right] \tag{8.103}$$

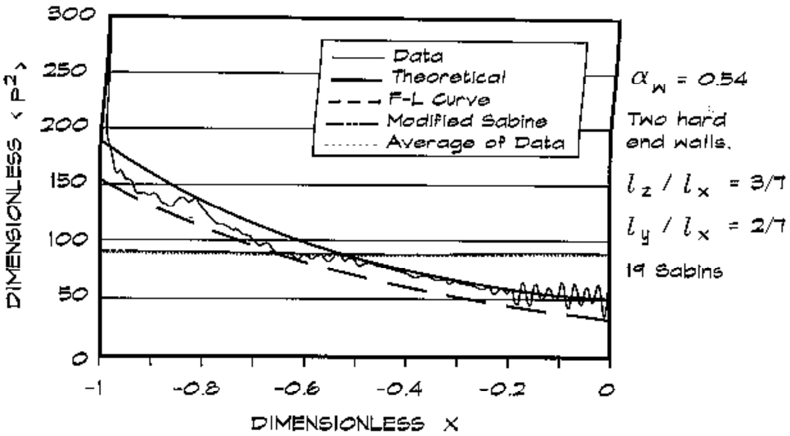
where $\gamma = \alpha_w\,\beta\,l_p/\left[(2-\alpha_w)\,S\right]$. Although this formula is somewhat more complicated than Eq. 8.91 it is still straightforward to use.

The result given by Eqs. 8.91 and 8.103 can be compared to more detailed calculations in Fig. 8.16. The agreement is good for both equations. Franzoni (2001) gives several other examples for different absorption coefficients, which also yield good agreement.

Reverberant Energy Balance in Long Narrow Rooms

An energy balance must still be maintained, where the energy produced by the source is absorbed by the materials in the room. In the Sabine theory, the balance is expressed as Eq. 8.83 and the reverberant field energy is assumed to be equally distributed throughout the room. In Franzoni's modified Sabine approach, the average reverberant field energy is the same as Sabine's, but the distribution is uneven. The average energy can be obtained either by integrating Eq. 8.103 over the length of the room or from the following arguments.

FIGURE 8.16 Comparison of Falloff Data—Empirical Fit and Theoretical (Franzoni, 2001)



The power removed from the room is

$$W_{out} = \sum_{\substack{\text{absorbing}\\ \text{surfaces, i}}} I_{\substack{\text{into}\\ \text{surface}}} \; \alpha_i \, S_i \tag{8.104}$$

The intensity incident on a surface is due to both the direct and reverberant-field components. From Eq. 8.94 the reverberant energy into a boundary surface is

$$I_r = \frac{\overline{p}^{\,2}_{incident}}{2\,\rho_0 c_0} = \frac{\overline{p}^{\,2}}{2\,\rho_0\,c_0}\frac{1}{(2-\alpha_i)} \tag{8.105}$$

and the average direct-field energy is

$$I_d = \frac{W_{in}}{S_{total}} \tag{8.106}$$

The power removed by the absorbing surfaces is

$$W_{out} = \sum_i \frac{\overline{p}^{\,2}}{2\,\rho_0\,c_0\,(2-\alpha_i)}\alpha_i\,S_i + \sum_i \frac{W_{in}}{S_{total}}\,\alpha_i\,S_i \tag{8.107}$$

which in terms of the average mean square pressure is the modified Sabine equation (Franzoni, 2001)

$$\overline{p}^{\,2}_{\substack{\text{spatial}\\ \text{average}}} = \sum_i \frac{4\,W_{in}\,\rho_0\,c_0}{\sum_i \alpha_i\,S_i/(1-\alpha_i/2)}\left(1 - \sum_i \alpha_i\,S_i/S_{total}\right) \tag{8.108}$$

When the same absorption coefficient applies to all surfaces this simplifies to

$$\bar{p}^2_{\substack{\text{spatial}\\\text{average}}} = \frac{4\,W_{in}\,\rho_0\,c_0}{A}(1-\alpha/2)\,(1-A/S_{total}) \tag{8.109}$$

The first term in the parentheses is a correction to the Sabine formula for the difference between the incoming and outgoing waves, and the second term is the power removed by the first reflection. Figure 8.16 also shows the results to be quite close to exact numerical simulations of the sound field.

Fine Structure of the Sound Decay

When an impulsive source such as a gunshot, bursting balloon, or electronically induced pulse excites a room with a brief impulsive sound, the room response contains a great deal of information about the acoustic properties of the space. First there is the initial sound decay in the first 10 to 20 msec of drop after the initial burst. The reverberation time based on this region is called the early decay time (EDT) and it is the time we react to. After the first impulse there is a string of pulses, which are the reflections from surfaces nearest the source and receiver. Thereafter follows a complicated train of pulses, which are the first few orders of reflections from the room surfaces. In this region the acoustical defects present in the room begin to appear. Long-delayed reflections show up as isolated pulses. Flutter echoes appear as repeated reflections that do not die out as quickly as the normal reverberant tail. Focusing can cause sound concentrations, which increase the reflected sound above the initial impulse. If the energy-time behavior of the room is filtered, it can be used to explore regions where modal patterns have formed and can contribute to coloration. A typical graph is shown in Fig. 8.17.

When two rooms are acoustically coupled the reverberation pattern in one room affects the sound in the other. When one has a longer reverberation time it may lead to a dual-slope reverberation pattern in the other. Consequently it is good practice to match the decay patterns of adjacent rooms unless it is the purpose to use one to augment the reverberant tail of the other.

FIGURE 8.17 Energy vs Time for an Impulsive Source

Sound pressure level at a point in a room for an impulsive sound. The direct sound arrives first followed by discrete reflections separated in time. Multiple reflections merge to become the reverberant field.