Energy-based vibroacoustics: SEA and beyond

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Introduction

Today, a number of possible approaches are available for the analysis of vibroacoustic problems. To find a required result with minimum effort and cost, the choice of the right method is very important. Here, an overview about one class of approaches shall be given. These approaches may be characterised by the term "energy-based". In contrast to the classical analysis of vibration, that is based on quantities such as force and displacement, these approaches are based on energy quantities (energy, energy density, power, ...). It is in principle possible to achieve the same information using either one or the other approach. But, depending on the application, energy-based methods may have some advantages. Most notably,

- the power - energy relation is not so sensitive to small parameter changes,
- energy quantities can be averaged more easily,
- most often, the actual goal of a computation are energy quantities (e.g. the sound pressure level that is an energy quantity).

To illustrate the last two statements, consider the simple problem (Table 1) that may be found in similar form in most textbooks on acoustics. Of both possible solutions, the second one is more elegant as only the information wanted is produced.

Problem:
Compute the sound pressure level (SPL) in a cavity for given excitation and absorption.
Solution 1:
- set up wave equation
- apply appropriate boundary conditions
- solve the equation using an analytical or a numerical method
- result: sound pressure distribution
- average to reduce unnecessary information overhead
- result: mean SPL $L_{p,m}$ in cavity

Solution 2: (energy-based)
- calculate input power (if not already given)
- use: $L_{p,m} = L_w - 10\lg\frac{A}{A_0}$
- note that result is an *expected value*

Table 1: An illustration of different approaches

Statistical energy analysis

Today the statistical energy analysis (SEA) is the most famous energy-based method. Although similar approaches were used before in room acoustics, the actual development of SEA started in the early 1960 with the application to vibroacoustic problems in aerospace engineering. "Statistical" means, that the variables are drawn from statistical population and all results are expected values. "Energy" denotes that energy variables are used and, according to Lyon[1], "Analysis" means here, that SEA is more a general approach rather than a particular technique.

The main idea in SEA is that a structure is partitioned into coupled "subsystems" and the stored and exchanged energies are analysed. The original theory is based on the study of interaction of groups of modes. While this concept is clever from a mathematical point of view, it is not very practical to use it for a quick insight. Thus, in what follows mathematical developments are dropped for the benefit of those interested in an overview on SEA. The reader interested in more in-depth treatment is referred to a recent compilation[2] as starting point.

Some basics

Consider a single subsystem – a separated part of the structure that is to be analysed. Any excitation acting on the subsystem can be characterised by the resulting power input P_i into the subsystem (Figure 1, left). If power is injected, the subsystem stores vibrational energy W_i. In practice, there will be also a power loss P_{ii} (e.g. due to dissipation). This power loss may be related to the stored energy by the damping loss factor η_i by:

$$P_{ii} = \omega\eta_i W_i. \quad (1)$$

Further, if the analysis is restricted to steady state, it is clear that power input equals power loss: $P_i = P_{ii}$.

Consider now a second subsystem (Figure 1, right). If this subsystem were coupled to the first, the same power balance would hold for both subsystems i and j, respectively. As a result of the coupling the subsystems share their vibrational energies. Power flows from subsystem i to subsystem j. From the viewpoint of subsystem i, this power flow P_{ij} is a power loss. A power flow exists also in the reverse direction (P_{ji}) that is as a result a power gain for subsystem j. To characterise these power flows, a special quantity, the coupling loss factor η_{ij}, is used in SEA. It is defined similarly to the damping loss factor in (1):

$$P_{ij} = \omega\eta_{ij} W_i. \quad (2)$$

In Figure 2, the case of four subsystems is shown as an example. This time, there is only a power input to subsystem 1. This power input is equal to the sum power

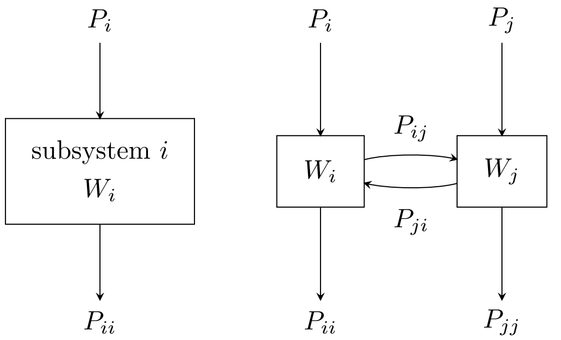

Figure 1: left: A single subsystem; **right:** coupled subsystems

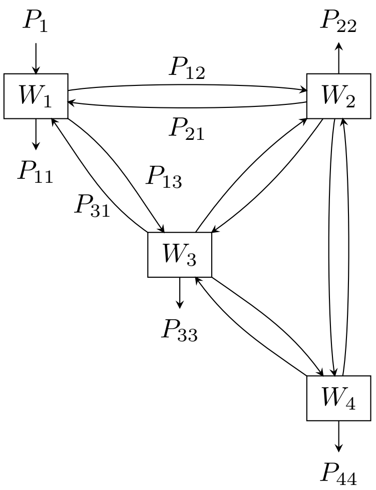

Figure 2: Example of four coupled subsystems

losses (due to dissipation and coupling) for that subsystem minus the power gains coming from the subsystems 2 and 3. The power balance reads then:

$$P_1 = P_{11} + P_{12} + P_{13} - P_{21} - P_{31}. \tag{3}$$

Similar power balances may be established for the remaining subsystems:

$$0 = P_{22} + P_{21} + P_{23} + P_{24} - P_{12} - P_{32} - P_{42}, \tag{4}$$
$$0 = P_{33} + P_{31} + P_{32} + P_{34} - P_{13} - P_{23} - P_{43}, \tag{5}$$
$$0 = P_{44} + P_{42} + P_{43} - P_{24} - P_{34}. \tag{6}$$

As there is no power input into these subsystems, the left hand side vanishes in the equations.

The power quantities in (3)-(6) may be substituted by the damping and coupling loss factors and the vibrational energy as in (1) and (2). The resulting equations may then be written down together as a system of equations:

$$\omega \begin{pmatrix} \eta_{11} & -\eta_{21} & -\eta_{31} & -\eta_{41} \\ -\eta_{12} & \eta_{22} & -\eta_{32} & -\eta_{42} \\ -\eta_{13} & -\eta_{23} & \eta_{33} & -\eta_{43} \\ -\eta_{14} & -\eta_{24} & -\eta_{34} & \eta_{44} \end{pmatrix} \begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \end{pmatrix} = \begin{pmatrix} P_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{7}$$

In the example, subsystems 1 and 4 are not coupled and consequently the corresponding coupling loss factors are zero. The diagonal elements of the loss factor matrix in (7) are called the total loss factors as they are the sum of all coupling loss factors that are associated with power losses for the respective subsystem:

$$\eta_{ii} = \eta_i + \sum_{j,j\neq i} \eta_{ij}. \tag{8}$$

The loss factor matrix in (7) can be rendered symmetric by introducing the modal densities (number of modes or resonances per frequency band) of the subsystems.

What is a subsystem?

So far, no detailed definition for a subsystem was given here. A subsystem can be seen as a part or physical element of the structure ("the system") that is to be analysed. To be modelled as a subsystem that part or element must be capable of vibrating *quite* independently from other elements. The word "quite" is emphasised here because as long as the element is not separated from the rest of the structure its vibration is not truly independent. Next, it is required for a subsystem to vibrate in resonant mode. That means, if the excitation is suddenly switched off, the vibrational energy stored in the subsystem will decay rather then drop to zero immediately (a point mass for instance is no suitable candidate for a subsystem). Thus, a reverberant sound field exists within the subsystem. If different wave types exist in the element, each of the corresponding sound fields is modelled by one subsystem. It should be noted in passing that the original definition for a subsystem makes the mathematical treatment easier: a subsystem is a group of "similar" energy storage modes.

To illustrate this definition, some examples of vibro/-acoustic elements that may be treated as subsystems shall be given here together with the necessary input data to characterise them:

- an acoustic cavity (a room): longitudinal waves – only one subsystem needed, characterised by volume, fluid parameters, absorption
- a plate: bending, compression and shear waves modelled by three subsystems, characterised by area, thickness, material parameters, damping
- a beam: four wave types – four subsystems, characterised by length, shape of cross-section, material parameters, damping
- shells, non-isotropic plates, ...

It is worth to notice that the energy stored in the subsystem is related to measurable quantities such as sound pressure level. This fact enables the practical application of the purely energy-based SEA equations.

The statistics in statistical energy analysis

Tough SEA is called statistical, no explicit statistics can be seen in the above SEA equations. For the simple theoretical approach taken here, statistical operations consist of a threefold average that is more implicit. First,

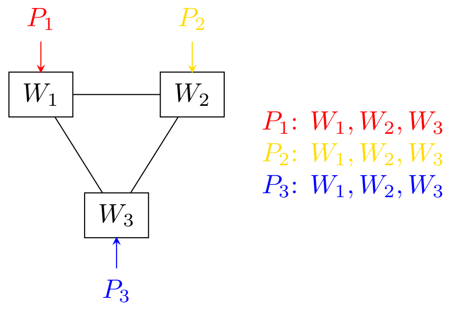

Figure 3: Procedure of experimental SEA: power is injected to each subsystem in turn

although not mentioned yet, calculation is done always for a frequency band (octave, third-octave, ...), but only at the centre frequency. This is an average of frequency. Next, only one variable is used to characterise the energy in one subsystem. This corresponds to a spatial average of the subsystem. Finally, by using very few parameters to characterise a subsystem there is no possibility to restore all information from these parameters that is necessary to describe the vibrational behaviour in detail. For instance, only area, thickness, material parameters are used as input parameters for a plate. Consequently, as long as these parameters remain the same, the shape of the plate does not matter in SEA. A circular, a quadratic or a trapezoidal plate map all to the same SEA model. This kind of average is called ensemble average. In modal space, all three averages are included in the average over a group of modes. It shall be noted in passing that besides the estimation of mean or expected values from the averages the estimation of variance is also possible, but is not staightforward. Generally, the variance is acceptable only at mid to high frequencies. That is why SEA is often referred to as a high-frequency method.

Procedures of experimental and predictive SEA

In order to apply SEA, an SEA model must be prepared. The most important step in doing so is to break up the structure, which should be analysed, into subsystems. This is often difficult to perform because the underlying theoretical assumptions have to be met as good as possible and at the same time a number of practical considerations must be taken into account. Moreover, for a given structure there are several models possible normally, so the modelling is ambiguous. After deciding for a model, the frequency range and band widths of interest must be identified. The mentioned steps are common to the two modes of SEA which should be detailed in the following.

In experimental SEA the main idea is to determine all quantities in (7) by experiment. To this end the following procedure is implemented: Power is injected to each subsystem in the structure in turn. This may be done for instance by means of a hammer, a shaker or a loudspeaker. Then, each time the energy in each subsystem is measured (by accelerometers or microphones). As depicted in Figure 3, for each subsystem a set of energies in now available and the following equation can be set up using (7):

$$\begin{pmatrix} P_1 & 0 & 0 \\ 0 & P_2 & 0 \\ 0 & 0 & P_3 \end{pmatrix} = \omega \begin{pmatrix} \eta_{11} & -\eta_{21} & -\eta_{31} \\ -\eta_{12} & \eta_{22} & -\eta_{32} \\ -\eta_{13} & -\eta_{23} & \eta_{33} \end{pmatrix} \begin{pmatrix} W_1 & W_1 & W_1 \\ W_2 & W_2 & W_2 \\ W_3 & W_3 & W_3 \end{pmatrix}. \quad (9)$$

The colours here correspond to those in Figure 3. By inverting the matrix of energies, this system may be solved to get the coupling and damping loss factors. In practice the matrix of energies is often bad conditioned but a number of methods have been developed to cope with that. Knowing now the matrix of loss factors, main paths of power may be identified, the effect of modifications may be assessed and a sensitivity analysis may be performed to find those factors that are most important for a given transmission scenario.

The basic idea in predictive SEA is to assess the coupling loss factors theoretically. Thus, it is possible to predict the behaviour of a structure even in an early stage of its design when no object is available for measurements. The procedure is as follows: First, the damping loss factors are estimated either from measurement, from tables, from calculations or simply from "experience". Then, the input power has to be determined (by experiment or calculation). Alternatively, the input power is set to unity, if only transmission loss is of interest and not absolute response values. The coupling loss factors may be calculated or assessed by several different techniques depending on the specific case of the coupling:

- from radiation or transmission efficiencies (wave approach)
- using modal approaches
- using numerical methods (e.g. Finite Element Method)
- coupling power proportionality $\eta_{ij} = \eta_{ji} n_j / n_i$ (n_i, n_j are the modal densities of subsystems i and j)
- ...

With all necessary input parameters available, the SEA equations may be solved and the response of the structure may be predicted. As in experimental SEA this enables a number of useful possibilities for analysis.

In some cases, it is useful to mix experimental and predictive SEA. However, great care must be taken as due to the ambiguous modelling this will not always lead to the expected results.

Applications of SEA

SEA is applicable for a great number of different problems. In many cases it is the only alternative left that is able to deal with high-frequency vibro-acoustic problems.

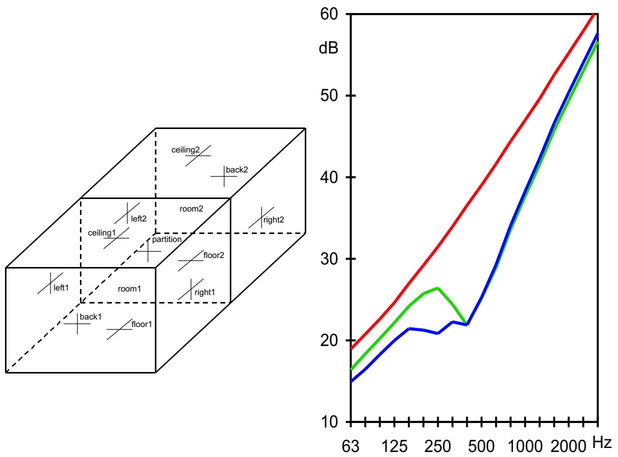

Figure 4: left: Two rooms with side walls (light concrete, 15 cm), floor and ceiling (heavy concrete, 19 cm) separated by a partition wall (light concrete, 10 cm), **right:** SPL difference between the two rooms: non-resonant transmission only (red), non-resonant and resonant (green), +flanking transmission (blue)

Its use as a tool for quantifying (dominant) transmission paths is also very efficient. To name a few areas of application for SEA, the following should be mentioned: building acoustics (handle flanking transmission !), vehicle interior noise (also for railway and ships), sound package modelling (efficient treatment of multi-path airborne sound transmission and absorption), (turbulence boundary layer induced) vibration of aircraft and launch vehicles.

To further illustrate the use of SEA, some examples shall be given here. The first example[3] is concerned with flanking transmission in building acoustics. In Figure 4, a sketch of a double room arrangement with a partition between the rooms is shown. This maps to an SEA model with 35 subsystems, two for the two rooms and three for the partition and for each of the walls, ceilings and floors. Three different stages of the model can be used for calculation:

- two subsystems (both rooms), only non-resonant transmission (mass law) is considered
- three subsystems (both rooms and the bending waves in the partition), non-resonant and resonant transmission
- all 35 subsystems, non-resonant, resonant and flanking transmission

The typical dip around the critical frequency of the partition (400 Hz) is clearly notable in the second case, see Figure 4. In the last case the influence of the different critical frequency of the flanking wall manifests itself through the additional dip around that frequency (250 Hz).

Another example is the prediction of structure-borne sound transmission in an untrimmed car body shell. This time the model consists of 280 subsystems. In Figure 5 the beam and the plate subsystems are shown. The model is used to predict the effect that modifications of the body have to the transmission of sound from an excitation at the engine mounts into the passenger compartment. The modifications used are intended primarily to show the abilities of the method and consist of adding distributed mass to firewall, floor and roof in turn. Thus, it is possible to measure the change of transmission behaviour easily. In Figure 6, both measured and predicted results are shown. While the prediction is good for the case of the firewall and the roof, it seems to fail for the floor. This may be due to insufficient modelling of the space beneath the car. Typical for SEA, it is not easy to shed light on this because there are a number of possible reasons.

The last example is intended to demonstrate one of the real powers of SEA: to get an estimate very quickly. Consider the following problem: The wall thickness of the outer housing of a pump is to be changed from 4 mm steel to 1.5 mm steel. Assess the impact on the radiated sound (only structure-borne excitation needs to be taken into account). Figure 7, left shows the simple SEA model which can be set up and solved within minutes. The result (Figure 7), right shows that although the vibration level increases, due to shift of the critical frequency the radiated sound is less in a broad frequency range.

Advanced energy based methods

The main motivation for advanced methods is that besides its many advantages SEA has a number of problems and limitations. Without going to much into detail, a few of them should be named here. First, SEA theory requires a number of implicit assumptions to be valid, that are often not easy to meet in practice: weak coupling (this refers to the problem explained in the section on subsystems), damping should be not too low but also not too high, homogeneity of the subsystems is necessary to render the calculation of vibrational level from the energy valid, sound fields have to be reverberant *and* diffuse, etc. Next, as may be guessed from the car body example, SEA often requires high modelling expertise. Moreover, SEA delivers no information on local distribu-

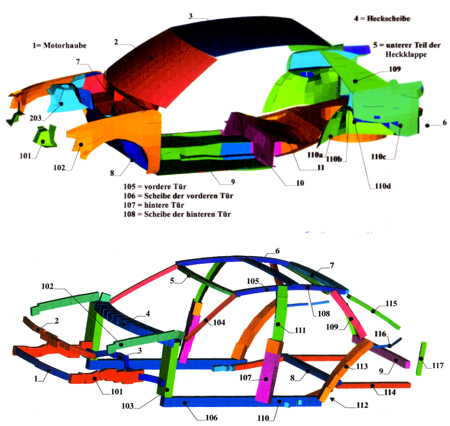

Figure 5: Graphical representation of an SEA model of a car body shell: plate (top) and beam (bottom) subsystems

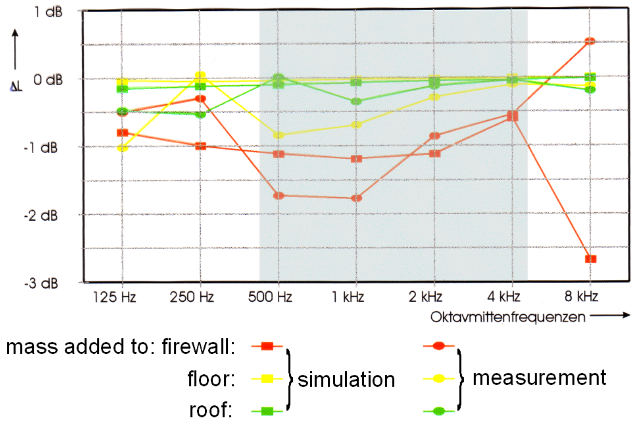

Figure 6: Predicted and measured result of modifications

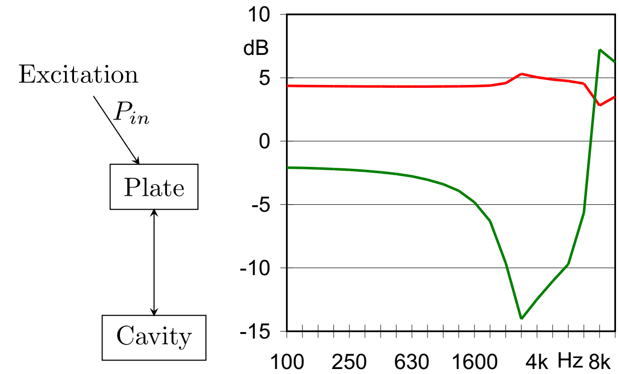

Figure 7: SEA model for the pump housing (left), result (right): vibration level (red) and sound pressure level (green) changes

tion vibration level within the subsystems (this becomes a problem, if the variance in the spatial distribution is to high). To close this list, the modelling approach is incompatible to classical FEM / BEM which makes the practical application of SEA in some cases too costly.

A great number of methods that circumvent some of these limitations and problems have been developed, but up to now only on an academic level. The following list gives an (incomplete) overview together with the main idea behind each method:

- Wave Intensity Analysis (WIA)[4]: like SEA, but fourier decomposition of wave field intensity in a subsystem - relaxes the diffuse field assumption of SEA
- Energy Finite Element Method (EFEM)[5]: analogy with heat conduction, see below
- (Integral) Smooth Energy Model (SEM)[6] or High Frequency Boundary Element Method (HFBEM): boundary integral formulation using energy variables, see below
- Energetic Mean Mobility Approach (EMMA)[7]: suited for treatment of heterogenous structures, especially for experimental application
- Complex Envelope Distribution Analysis (CEDA)[8]: analysis using a cepstrum calculated from wavenumber spectrum rather than from frequency spectrum (not energy-based)
- Hybrid Methods: incorporate explicit "modal" behaviour of components into SEA-like models, e.g.[9]

Both the energy finite element method and the high frequency boundary element method shall be explained here in little more detail. They both start with the same fundamental assumption that the principle of energy conservation may be used to formulate an equation of energy continuity. In contrast to SEA not a subsystem but an infinitesimal volume is considered. The amount of vibrational energy stored in that volume will be governed by a) power losses P_{diss} (due to dissipation), b) power injection P_{in} (due to external load) and c) energy transport $c_g W$ through the volume boundaries (energy flow per unit area). Thus, an equation for the time-derivative of the energy may be set up:

$$\frac{\partial}{\partial t} W = -\nabla \cdot (\mathbf{c_g} W) + P_{in} - P_{diss}, \qquad (10)$$

which can be seen as an energy continuity equation. Reformulated using power density and energy density, assume steady state and substitute dissipated power with the damping loss factor the equation reads:

$$p_{in} = \nabla \cdot \mathbf{I} + \omega\eta w. \qquad (11)$$

This equation may used as a basis for calculating the spatial distribution of energy density w from the known input power density p_{in}. Two different approaches exist to provide the necessary relation between intensity $\mathbf{I}$ and energy density needed to transform (11) into a solvable equation.

The first approach assumes that the wave field consists of superposed plane waves only:

$$\mathbf{I} = -\frac{c_g^2}{\omega\eta} \nabla w. \qquad (12)$$

If substituted into (11), the resulting equation is analog to the equation for heat conduction in a plate including convection:

$$p_{in} = -\frac{c_g^2}{\omega\eta} \Delta w + \omega\eta w, \qquad (13)$$

$$P_{in} = -\lambda \Delta T + \alpha \left(T - T_B\right). \qquad (14)$$

Thus, a vibration conduction coefficient $\frac{c_g^2}{\omega\eta}$ as well as a vibration convection coefficient $\omega\eta$ may be defined. (13) may be solved using finite element (FE) techniques (thus the name EFEM). In particular, using the both coefficients as input parameters, existing FE codes for heat conduction may be used for the calculation. While this is an advantage, the method possesses a serious problem: because the assumption of plane does not hold for wave fields dominated by spherical waves, it is not valid in these cases. However, it is not fully clear how relevant this is for practical application.

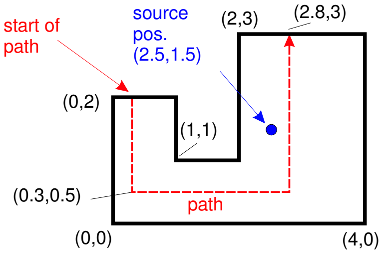

Figure 8: Large steel plate; path is used to plot the result in Figure 9

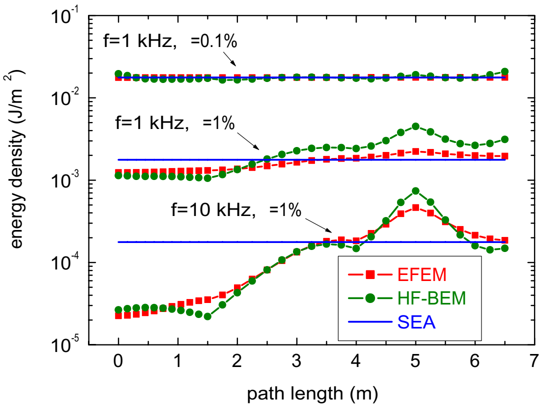

Figure 9: SEA, EFEM and HFBEM results for the plate

The second approach, that results in the HFBEM, assumes that the wave field may be synthesised by spherical waves from sources together with spherical waves with origins at the boundary of the wave field. As a consequence, the energy density at some location M is given by integration over all (primary) sources S and over all boundary sources Q:

$$w(M) = \int_{\Omega} \rho(S) G(S, M) dS + \int_{\partial\Omega} \sigma(Q) f(\mathbf{u}_{MQ}, \vec{n}_Q) G(Q, M) dQ. \quad (15)$$

Without going into much detail, it shall be pointed out that from this, an integral equation for the unknown source strengths σ of the sources at the boundary may be formulated:

$$\sigma(Q) = \frac{\varrho \gamma_n}{\gamma} \left(\int_{\Omega} \rho(S) \mathbf{H}(S, Q) dS + \int_{\partial\Omega} \sigma(Q') f(\mathbf{u}_{QQ'}, \vec{n}_{Q'}) \mathbf{H}(Q', Q) dQ' \right) \cdot \vec{n}_Q. \quad (16)$$

This equation can be solved by a boundary element approach using a collocation technique. That means, the continuous function σ is approximated by a stepwise constant function and the second integral in (16) becomes a sum. Thus, the equation can be transferred into a linear system of equations. Once this system is solved for the source strengths, the spatial energy density distribution can be calculated using the discretised form of (15).

A last example[10] shall demonstrate the capabilities of both EFEM and HFBEM. A large steel plate (Figure 8) with a single point source (e.g. a shaker) is considered. The results from SEA, EFEM and HFBEM are shown in Figure 9 for different frequencies and damping. They are plotted along the path shown in Figure 8. While the SEA provides no information about the spatial distribution of the energy density, the other methods show a somewhat higher energy level in the vicinity of the source ($\approx$5 m). For higher damping, EFEM fails to predict the direct field correctly. The HFBEM results are less reliable for energy equipartition in the low damping case.

Summary

In this paper, a short overview was given on the basic ideas behind methods for the treatment of vibroacoustic problems that are based on energy variables. In particular, theory and application of the most popular of these methods, the statistical energy analysis (SEA), were explained. Main advantages were shown as well as the problems and limitations. As example for methods that go beyond the limits of SEA, a brief introduction into energy finite element method (EFEM) and high frequency boundary element method (HFBEM) was given.

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