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Hivoss

Human induced Vibrations of Steel Structures

Vibration Design of Floors

Guideline

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Summary

Modern large span floors and light weight floors show a tendency to vibrate under service conditions. This guideline presents a method with which floors can be easily designed for these vibrations.

The scope of this guideline are floors in office and/or residential buildings that might be excited by persons walking normally and which can effect the comfort of other building users. The aim of the design guide is to help specify comfort requirements for occupants and to perform a design that guarantees the specified comfort.

This guideline is accompanied by a background document which also presents alternative and more general ways for the determination of the floor response to dynamic human induced forces.

The theoretical methods presented here and in the background document have been elaborated/investigated in the RFCS-Project “Vibration of Floors”. The guideline and background document are here disseminated under the grant of the Research fund for Coal and Steel within the project “HIVOSS”.

Guidance for the determination of the relevant dynamic floor characteristics and application examples are given in the annexes of this document.

1. Introduction

1.1. General

Floor structures are designed for ultimate limit states and serviceability limit state criteria:

- Ultimate limit states are those related to strength and stability;
- Serviceability limit states are mainly related to vibrations and hence are governed by stiffness, masses, damping and the excitation mechanisms.

For slender floor structures, as made in steel or composite construction, serviceability criteria govern the design.

This guideline gives guidance for:

- Specification of tolerable vibration by the introduction of acceptance classes and
- Prediction of floor response due to human induced vibration with respect to the intended use of the building.

For the prediction of floor vibration several dynamic floor characteristics need to be determined. These characteristics and simplified methods for their determination are briefly described. Design examples are given in annex B of this guideline.

1.2. Scope

The procedure provided in this guideline provides a simplified method for determining and verifying floor designs for vibrations due to walking. The guideline focuses on simple methods, design tools and recommendations for the acceptance of vibration of floors which are caused by people during normal use. The given design and assessment methods for floor vibrations are related to human induced vibrations, mainly caused by walking under normal conditions. Machine induced vibrations or vibrations due to traffic etc. are not covered by this guideline.

The guideline should not be applied to pedestrian bridges or other structures which do not have a structural characteristic or a the characteristic of use comparable to floors in buildings.

The guideline focuses on the prediction and evaluation of vibration at the design level.

1.3. References

[1] European Commission – Technical Steel Research: *Generalisation of criteria for floor vibrations for industrial, office, residential and public building and gymnastic halls*, RFCS Report EUR 21972 EN, ISBN 92-79-01705-5, 2006, http://europa.eu.int

[2] Hugo Bachmann, Walter Ammann. *Vibration of Structures induced by Man and Machines*” IABSE-AIPC-IVBH, Zürich 1987, ISBN 3-85748-052-X

[3] Waarts, P. *Trillingen van vloeren door lopen: Richtlijn voor het voorspellen, meten en beoordelen*. SBR, September 2005.

[4] Smith, A.L., Hicks, S.J., Devine, P.J. *Design of Floors for Vibrations: A New Approach.* SCI Publication P354, Ascot, 2007.

[5] ISO 2631. *Mechanical Vibration and Shock, Evaluation of human exposure to whole-body vibration*. International Organization for Standardization.

[6] ISO 10371. *Bases for design of structures – Serviceability of buildings and walkways against vibrations*. International Organization for Standardization.

1.4. Definitions

The definitions given here are oriented on the application of this guideline.

Damping *D*	Damping is the energy dissipation of a vibrating system. The total damping consists of • material and structural damping • damping by furniture and finishing (e.g. false floor) • geometrical radiation (propagation of energy through the structure)
Modal mass M_{mod} = generalised mass	In many cases, a system with several degrees of freedom can be reduced to a system with a single degree of freedom with frequency: $$f = \frac{1}{2\pi}\sqrt{\frac{K_{\text{mod}}}{M_{\text{mod}}}}$$ where f is the natural frequency K_{mod} is the modal stiffness M_{mod} is the modal mass Thus the modal mass can be interpreted to be the mass activated in a specific mode shape. The determination of the modal mass is described in chapter 0.

Natural Frequency *f* = Eigen frequency	Every structure has its specific dynamic behaviour with regard to vibration mode shape and duration T[s] of a single oscillation. The frequency *f* is the reciprocal of the oscillation time T ($f=1/T$). The natural frequency is the frequency of a free oscillation without continuously being driven by an exciter. Each structure has as many natural frequencies and associated mode shapes as degrees of freedom. They are commonly sorted by the amount of energy that is activated by the oscillation. Therefore the first natural frequency is that on the lowest energy level and is thus the most likely to be activated. The equation for the natural frequency of a single degree of freedom system is $$f=\frac{1}{2\pi}\sqrt{\frac{K}{M}}$$ Where: K is the stiffness M is the mass
OS-RMS$_{90}$	One step RMS- value of the acceleration for a significant step covering the intensity of 90% of peoples' steps walking normally. OS: One step RMS: Root mean square = effective value, of the acceleration *a*: $$a_{RMS}=\sqrt{\frac{1}{T}\int_0^T a(t)^2\,dt}\approx\frac{a_{Peak}}{\sqrt{2}}$$ where: T is the investigated period of time.

1.5. Variables, units and symbols

a	*acceleration*	[m/s^2]
B	*width*	[m]
f, f_i	*natural frequency under consideration*	[Hz]
$\delta(x,y)$	*Deflection at location x,y*	[m]
D	*Damping (% of critical damping)*	[-]
D_1	*Structural damping*	[-]
D_2	*Damping from furniture*	[-]
D_3	*Damping from finishes*	[-]
l	*length*	[m]
K, k	*stiffness*	[N/m]
M_{mod}	*Modal mass*	[kg]
M_{total}	*Total mass*	[kg]
OS-RMS	*One step root mean square value of the effective velocity*	[-]
OS-RMS$_{90}$	*90 percentile of OS-RMS values*	[-]
T	*period (of oscillation)*	[s]
t	*time*	[s]
p	*distributed load (per unit length or per unit area)*	[kN/m] or [kN/m^2]
δ	*deflection*	[m]
μ	*mass distribution per unit length or per unit area*	[kg/m] or [kg/m^2]

2. Design of floors against vibrations

2.1. Design procedure

The design procedure described in this guideline corresponds to a simplified procedure with which a floor design can be verified for vibrations due to human walking loads. The first step in the procedure is to determine the basic floor characteristics or parameters. Using these parameters and a set of graphs, a quantity called the 90% one-step RMS value, which characterizes the floor response due to walking, is obtained. This value is then compared to recommended values for different floor destinations. These three steps are visualized in Figure 1. This method is referred to the simplified procedure or hand calculation method and is treated in chapter 4.

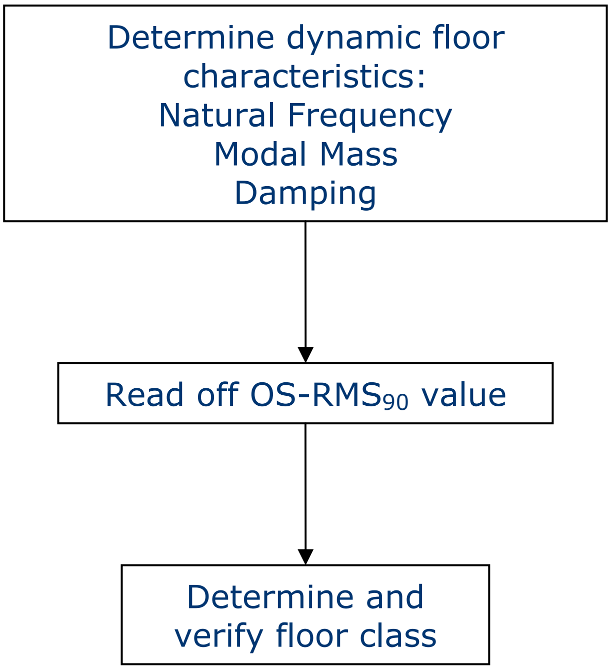


Figure 1: Design procedure.

2.2. Related design methods

2.2.1. Hand calculation method using measurements

The hand calculation method can also be used in those cases where the floor characteristics have been determined by experiment. For a discussion on how to extract dynamic floor characteristics from tests refer to [1] and [3].

2.2.2. Transfer function method

In the transfer function method, instead of using modal parameters such as modal mass stiffness and damping, the floor's characteristics are described in terms of a frequency response function, FRF, or transfer function. Starting from a statistical description of walking loads, a probabilistic analysis is carried out to determine the OS-RMS$_{90}$ value of the floor. The method described in more detail in [1] and [3].

In the hand calculation method, being a simplified version of the transfer function method, the classical transfer function for a single degree of freedom mass-spring-dashpot system is used and the probabilistic analysis to obtain the OS-RMS$_{90}$ has been carried out in advance.

The transfer function method can be applied where the floor response is obtained by finite element calculations or by measurements. The same set of acceptance criteria as described in this guideline are used.

2.2.3. Modal superposition

Two methods of analysis based on modal superposition are presented in the guideline by the Steel Construction Institute, [4]. The general method uses finite element software to determine the modal properties of the floor for a number of modes, and then applies design walking forces to the floor to determine a response, in terms of acceleration. The simplified method is based on a parametric study using the general approach, and is presented as analytical formulae. Unlike the hand calculation method, the approach used for the modal superposition method is deterministic and the results are comparable directly to limits supplied by international standards in [5] and [6]. This method also allows the influence of separation between the walking activity and the receiver to be considered, which enables the isolation of critical areas such as hospital operating theatres to be determined. Further information on this approach is given in the background document.

3. Classification of Vibrations

3.1. Quantity to be assessed

The perception of vibrations by persons and the associated annoyance depends on several aspects. The most important are:

- The direction of the vibration. In this guideline only vertical vibrations are considered.
- The posture of people such as standing, laying or sitting.
- The current activity of an occupant is of relevance for his or her perception of vibrations, for example, persons working in the production of a factory will perceive vibrations differently to those working in an office.
- Additionally, age and health of affected people may play a role in determining the level of annoyance perceived.

Thus the perception of vibrations varies between individuals and can only be judged in a way that fulfils the expectations of comfort for the majority of people.

It should be considered that the vibrations levels considered in this guideline are relevant for the comfort of the occupants only. They are not relevant for structural integrity.

Aiming at an universal assessment procedure for human induced vibration it is recommended to adopt the so-called one step RMS value (OS-RMS) as a measure for assessing floor vibrations. The OS-RMS values corresponds to the vibration caused by one relevant step onto the floor.

As the dynamic effect of people walking on a floor depends on several different factors, such as weight and speed of walking people, their shoes, flooring, etc., the 90% OS-RMS ($OS\text{-}RMS_{90}$) value is recommended as assessment value. This value is defined as the 90 percentile of all the OS-RMS values obtained for a set of loads representing all possible combinations of persons' weights and walking speeds.

3.2. Floor classes

The following table classifies vibrations into six floor classes (A to F) and gives also recommendations for the assignment of classes with respect to the function of the considered floor.

Table 1: Classification of floor response and recommendation for the application of classes

Class	OS-RMS$_{90}$		Function of Floor												
	Lower Limit	Upper Limit	Critical Workspace	Health	Education	Residential	Office	Meeting	Retail	Hotel	Prison	Industrial	Sport		
A	0.0	0.1	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩		
B	0.1	0.2	🟨	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩		
C	0.2	0.8	🟥	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩		
D	0.8	3.2	🟥	🟨	🟨	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩		
E	3.2	12.8	🟥	🟥	🟥	🟨	🟨	🟨	🟨	🟨	🟨	🟩	🟩		
F	12.8	51.2	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟨	🟨		

🟩 Recommended
🟨 Critical
🟥 Not recommended

Limits on vibration are also given in International Standard ISO 10137[6], which is referenced in the Eurocodes. These limits are reproduced in Table 2, with the equivalent OS-RMS$_{90}$ limit.

Table 2: Vibration limits specified by ISO 10137 for continuous vibration

Place	Time	Multiplying Factor	OS-RMS$_{90}$ equivalent
Critical working areas (e.g. hospital operating-theatres, precision laboratories, etc.)	Day	1	0.1
	Night	1	0.1
Residential (e.g. flats, homes, hospitals)	Day	2 to 4	0.2 to 0.4
	Night	1.4	0.14
Quiet office, open plan	Day	2	0.2
	Night	2	0.2
General office (e.g. schools, offices)	Day	4	0.4
	Night	4	0.4
Workshops	Day	8	0.8
	Night	8	0.8

It is considered that these limits are unnecessarily harsh, and testing on a number of subjects found the limits in Table 1 to be more appropriate (see [1]).

4. Hand calculation method

The hand calculation method assumes that the dynamic response of a floor can represented by a single degree of freedom system. The eigenfrequency, modal mass and damping can be obtained by calculation as set out in this chapter. As discussed previously in section 2.2.1, the modal parameters can be also obtained from a measurement. As this guideline is intended to be used for the design of new buildings, testing procedures are excluded from it.

4.1. Determination of eigenfrequency and modal mass

In practice, the determination of floor characteristics can be performed by simple calculation methods (analytical formulas) or by Finite Element Analysis (FEA).

In the determination of the dynamic floor characteristics also a realistic fraction of imposed load should be considered in the mass of the floor. Experienced values for residential and office building are 10% to 20% of the imposed load. For very light floors it is recommended to include also the mass of one person. A minimum representative person's mass of 30 kg is recommended.

4.1.1. Finite Element Analysis

Different finite element programs can perform dynamic calculations and offer tools for the determination of natural frequencies. Many programs also calculate the modal mass automatically in a frequency analysis.

As the element types, the modelling of damping and the output is program specific, only some general information can be given in this guideline concerning FEA.

If FEA is applied for the design of a floor with respect to the vibration behaviour, it should be considered that the FEA-model for this purpose may differ significantly to that used for ultimate limit state (ULS) design as only small deflections are expected due to vibration.

A typical example is the selection of boundary conditions in vibration analysis compared to ULS design. A connection which is assumed to be a hinged connection in ULS may be assumed to provide full moment connection in a vibration analysis.

For concrete the dynamic modulus of elasticity should be considered to be 10% higher than the static tangent modulus E_{cm}.

4.1.2. Analytical formulas

For calculations by hand, Annex A gives formulas for the determination of frequency and modal mass for isotropic plates, orthotropic plates and beams.

4.2. Determination of damping

Damping has a great influence on the vibration behaviour of a floor. Independently of the way of determining natural frequency and modal mass, damping values for vibrating systems can be determined using Table 3 for different structural materials, furniture and finishing. The system damping D is obtained by summing up the appropriate values for D_1 to D_3.

Table 3: Determination of damping

Type	**Damping (% of critical damping)**
Structural Damping D_1	
Wood	6%
Concrete	2%
Steel	1%
Steel-concrete	1%
Damping due to furniture D_2	
Traditional office for 1 to 3 persons with separation walls	2%
Paperless office	0%
Open plan office.	1%
Library	1%
Houses	1%
Schools	0%
Gymnastic	0%
Damping due to finishes D_3	
Ceiling under the floor	1%
Free floating floor	0%
Swimming screed	1%
Total Damping $D = D_1 + D_2 + D_3$	

4.3. Determination of the floor class

When modal mass and frequency are determined, the OS-RMS_{90} value as well as the assignment of the floor classes can be obtained with the diagrams given in section 4.4. The relevant diagram needs to be selected according to the damping characteristics of the floor in the condition of use (considering finishing and furniture).

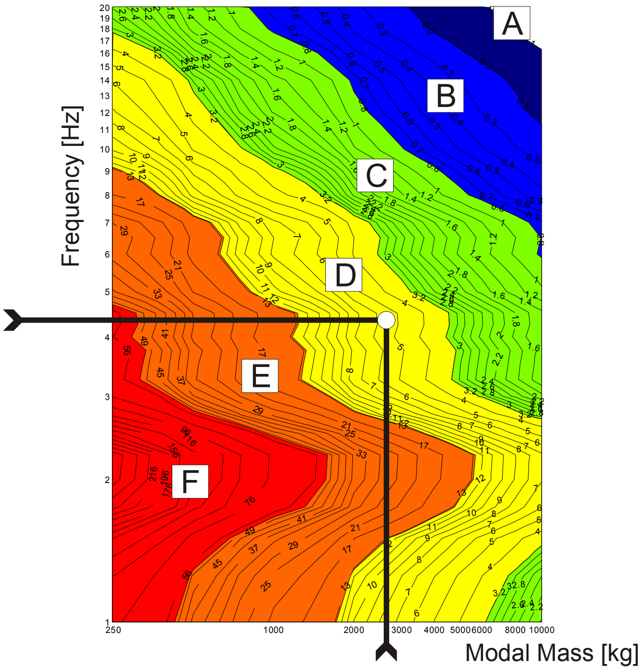


Figure 2: Application of diagrams

The diagram is applied by entering the x-axis with the modal mass and the y-axis with the corresponding eigenfrequency. The OS-RMS$_{90}$ value and the acceptance class can be read-off at the intersection of the lines extending from both entry points (see Figure 2).

4.3.1. Systems with more than one eigenfrequency

In some cases, the floor response may be characterized by more than one natural frequency. In these cases, the OS-RMS$_{90}$ must be determined as a combination of OS-RMS$_{90}$ values obtained for each mode of vibration. The procedure is as follows:

a) Determine the eigenfrequencies.

b) Determine the modal mass and damping corresponding to each eigenfrequency.

c) Determine the corresponding OS-RMS$_{90}$ value for each eigenfrequency.

d) Approximate the total (or combined) OS-RMS$_{90}$ value using:

$$\mathrm{OS-RMS}_{90} = \sqrt{\sum_i \mathrm{OS-RMS}_{90;i}{}^2}$$

e) Read off the corresponding floor class from Table 1.

4.4. OS-RMS$_{90}$ graphs for single degree of freedom systems

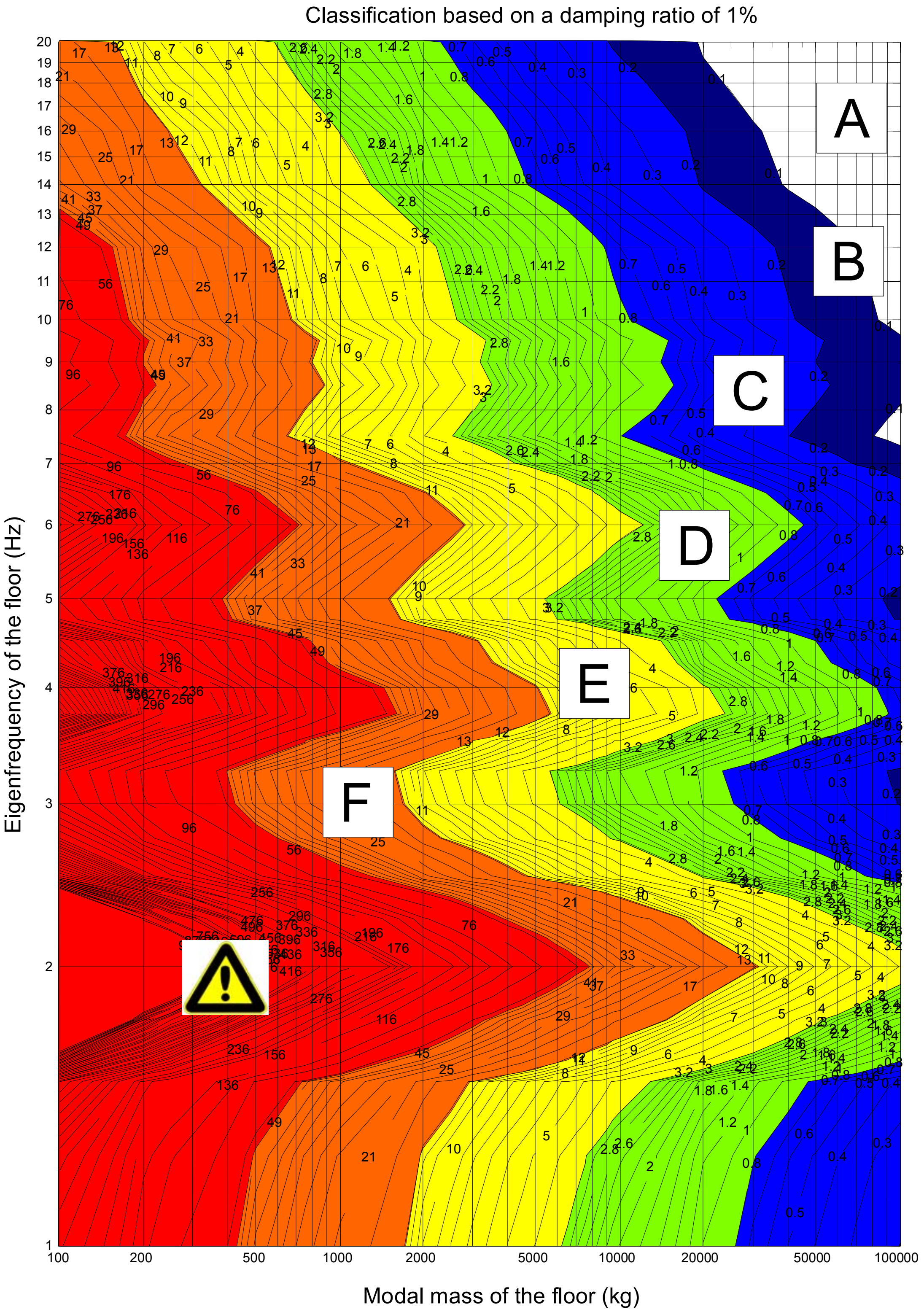


Figure 3: OS-RMS$_{90}$ for 1 % Damping

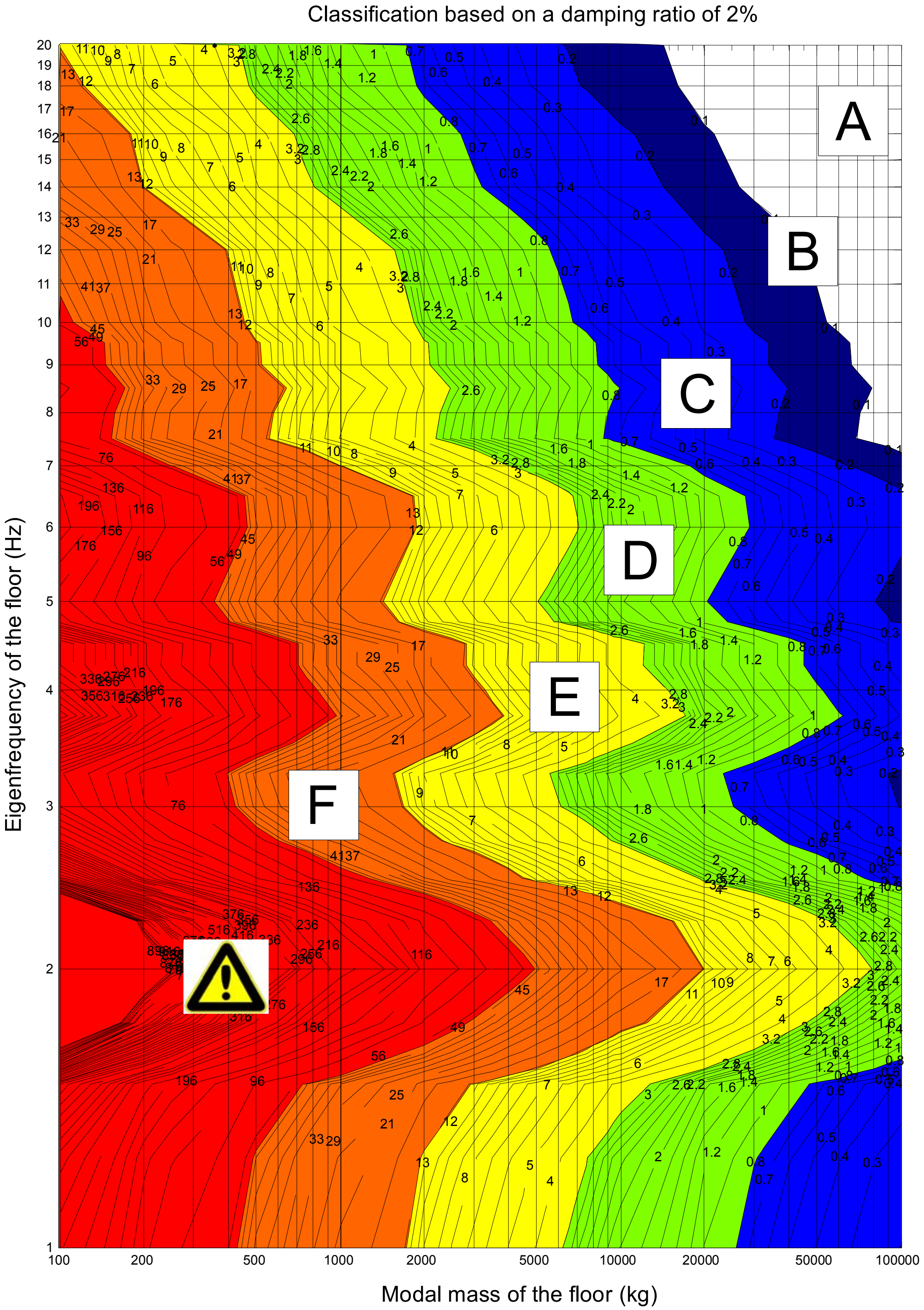


Figure 4: OS-RMS$_{90}$ for 2% Damping

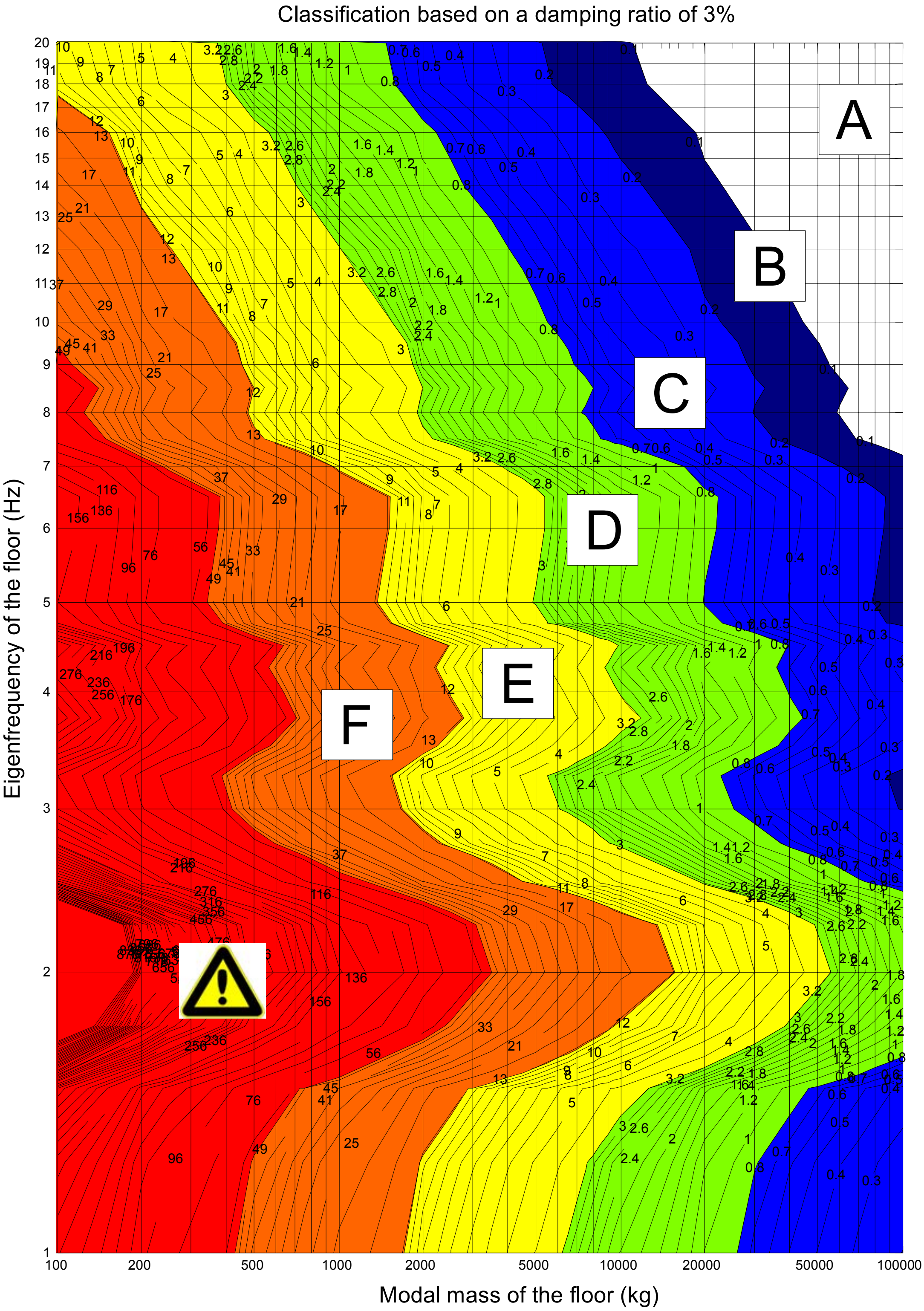


Figure 5: OS-RMS_{90} for 3% Damping

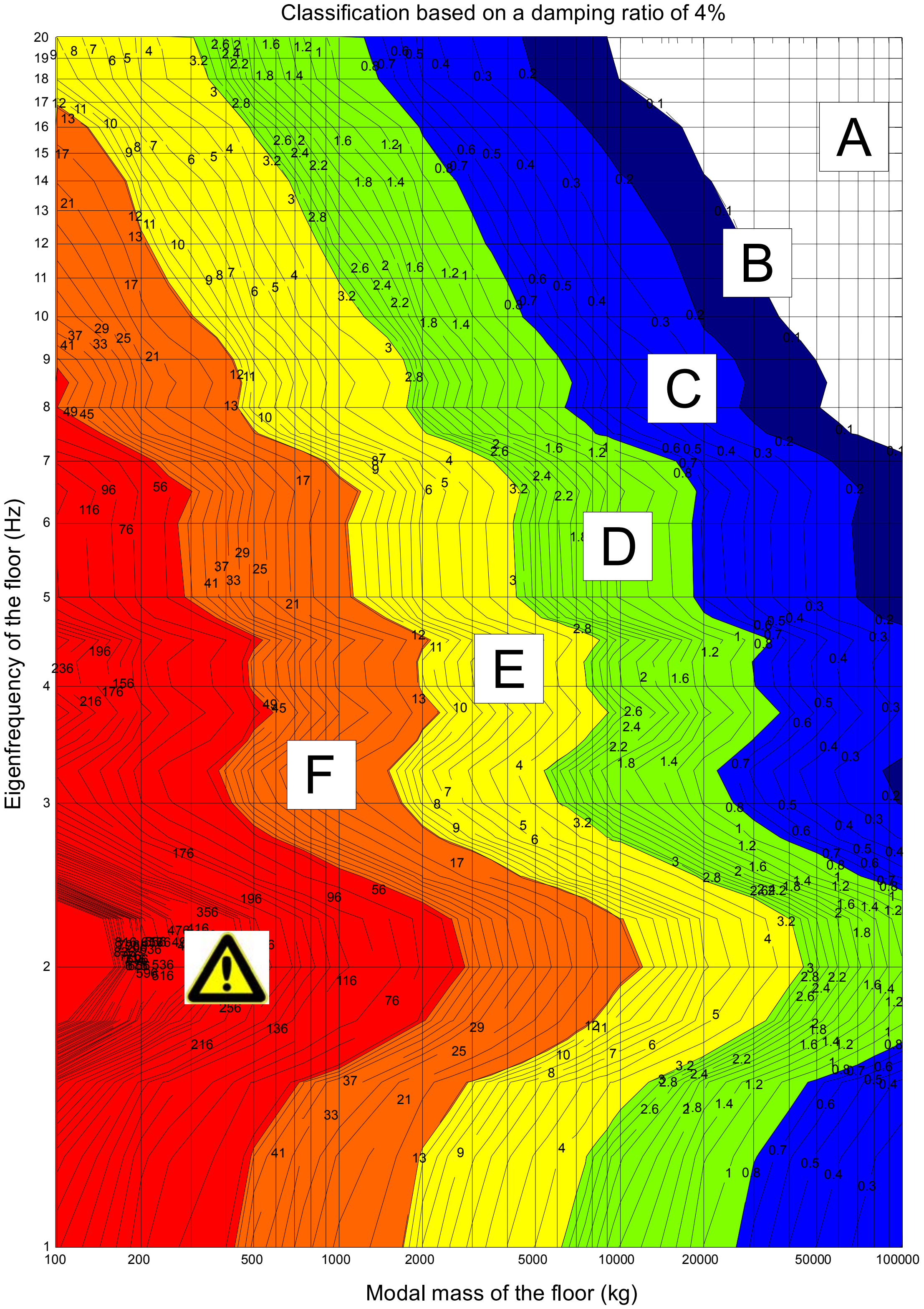

Classification based on a damping ratio of 4%

Figure 6: OS-RMS$_{90}$ for 4% Damping

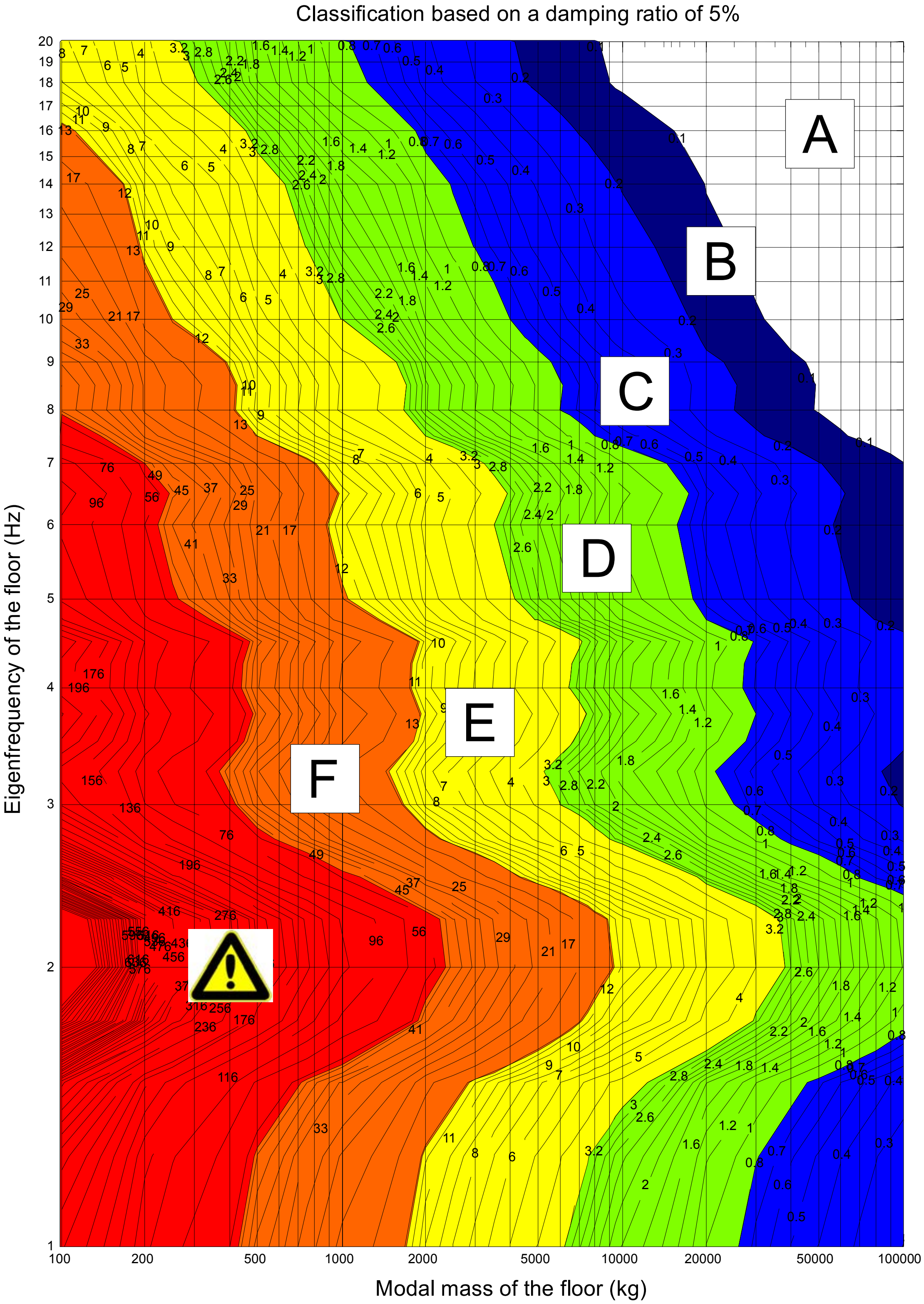


Figure 7: OS-RMS$_{90}$ for 5% Damping

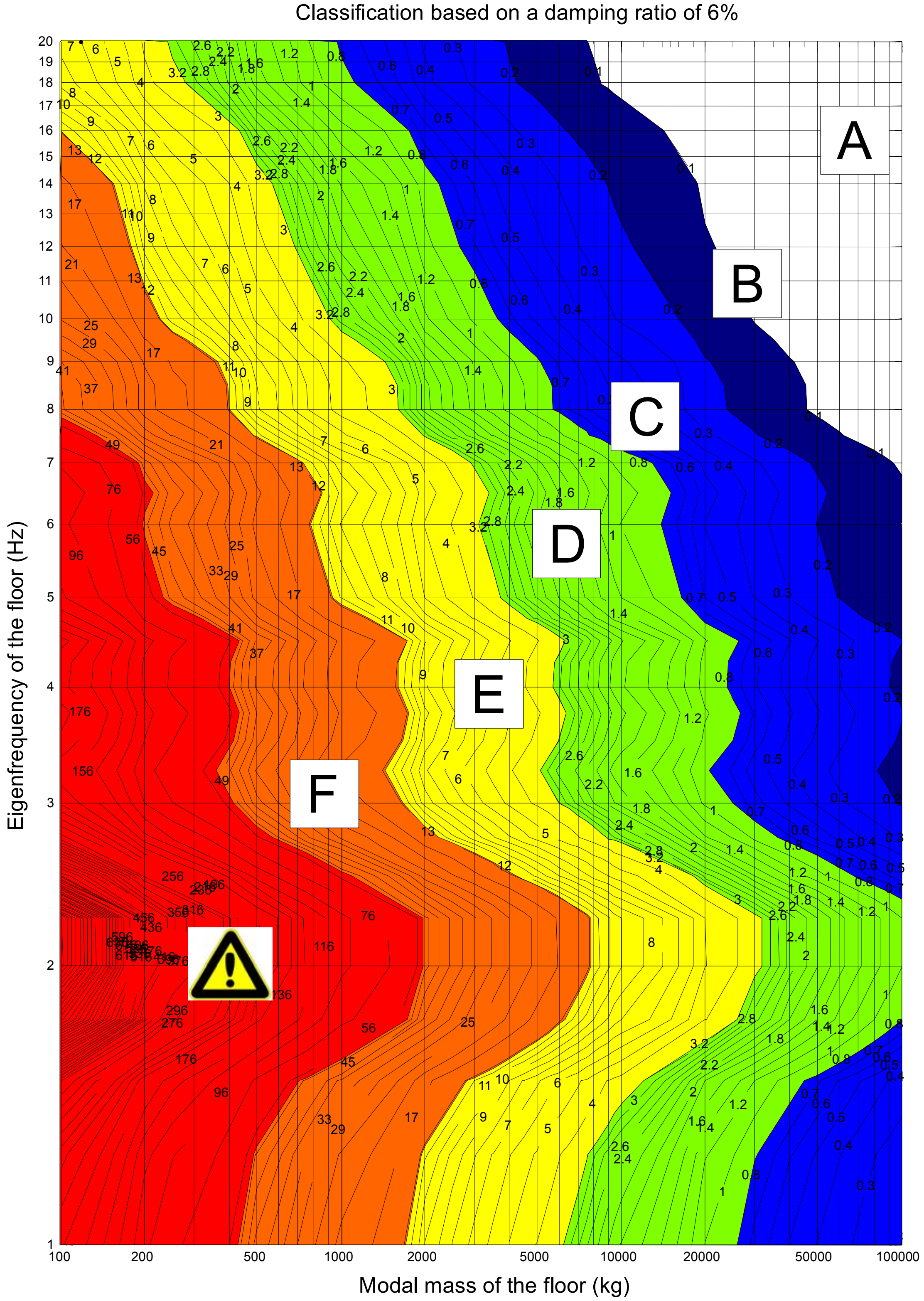


Figure 8:OS-RMS$_{90}$ for 6 % Damping

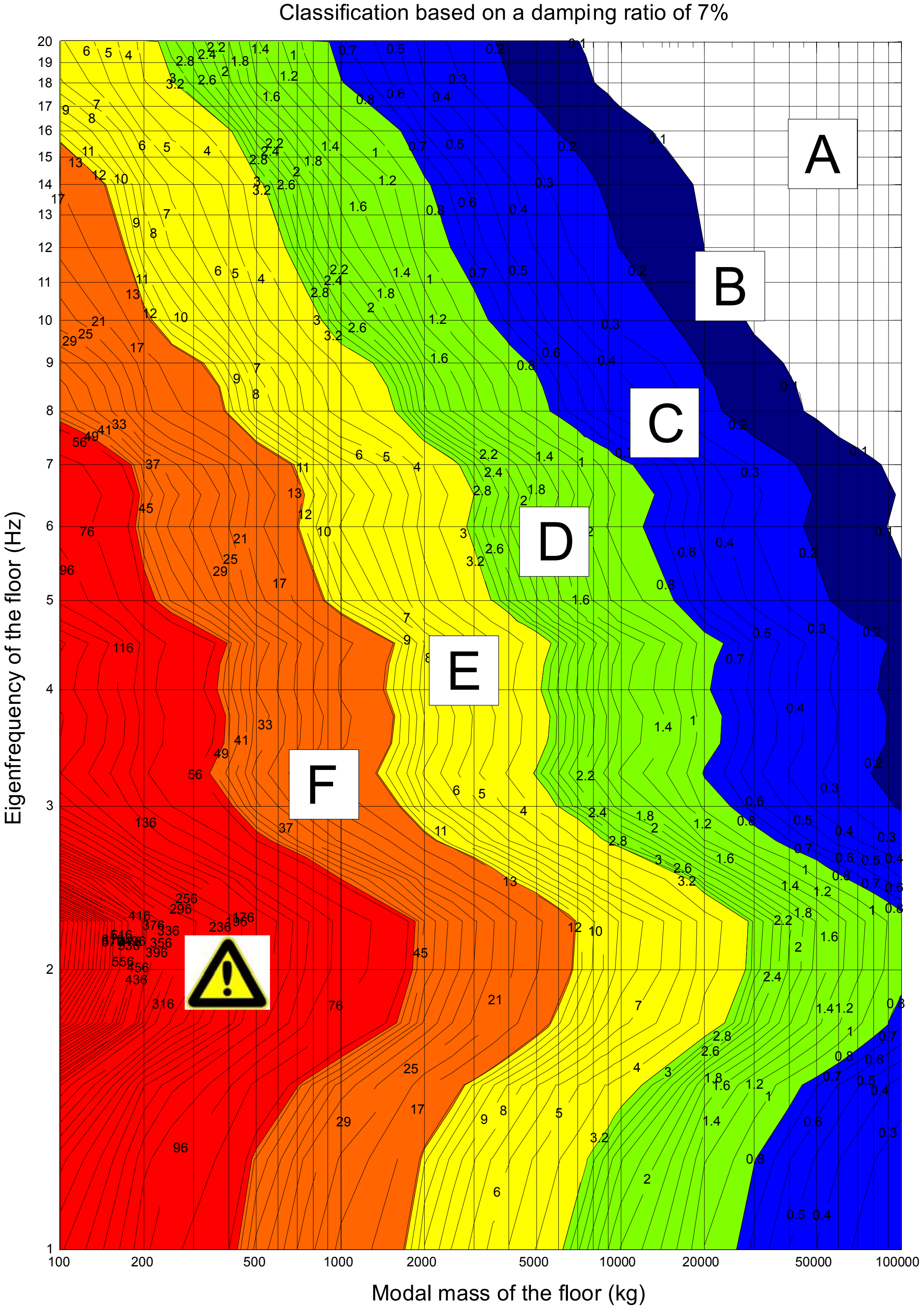


Figure 9: OS-RMS$_{90}$ for 7% Damping

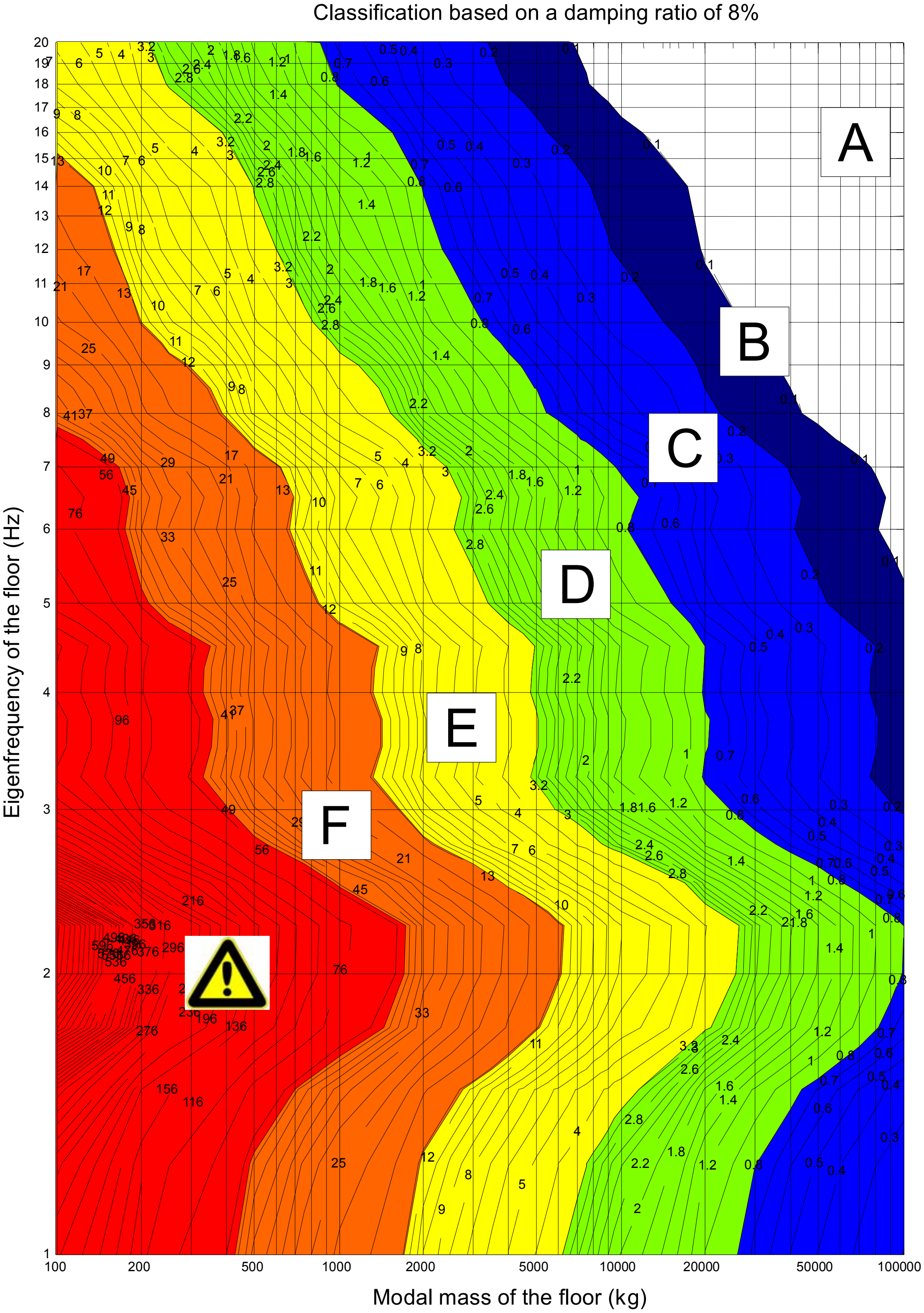
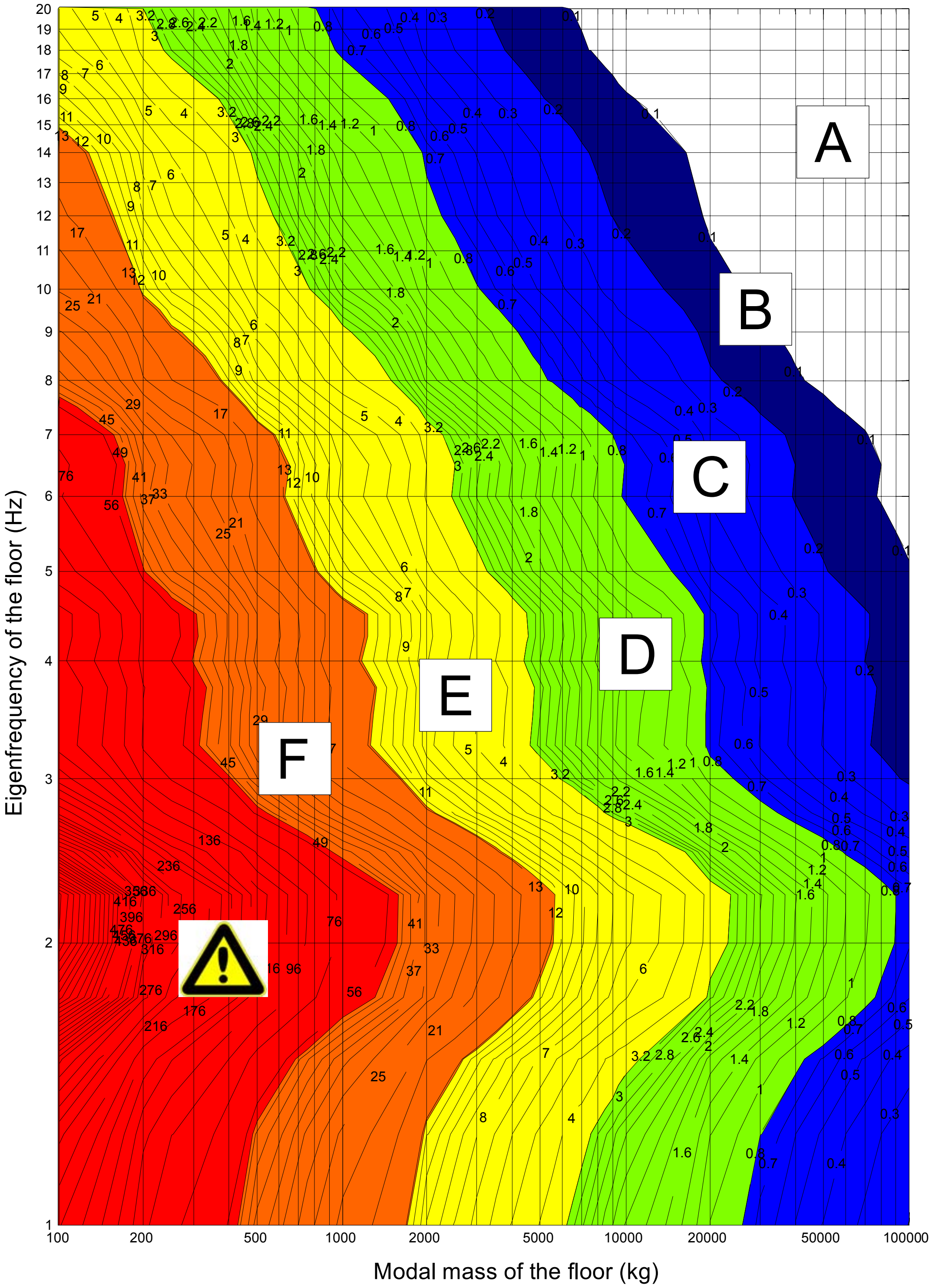


Figure 10: OS-RMS$_{90}$ for 8% Damping



Classification based on a damping ratio of 9%

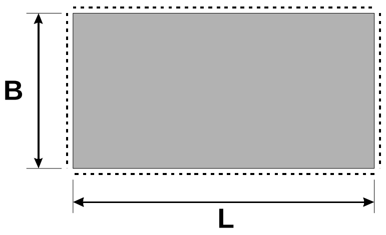
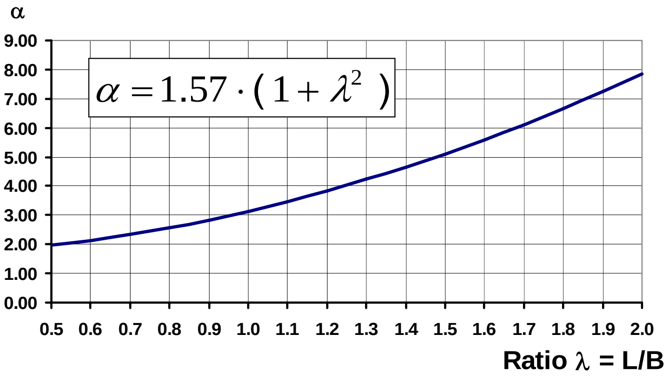
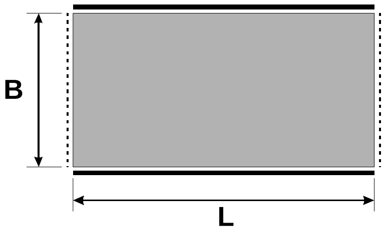
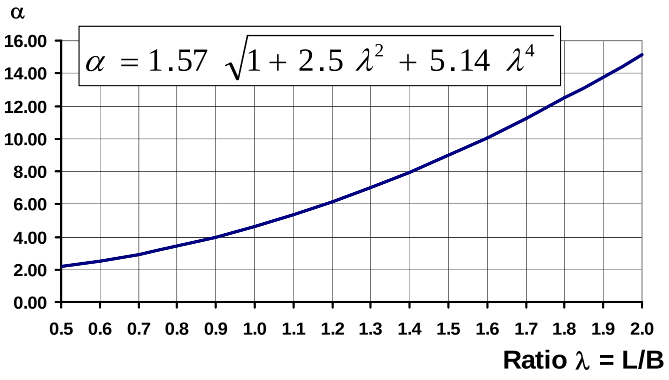
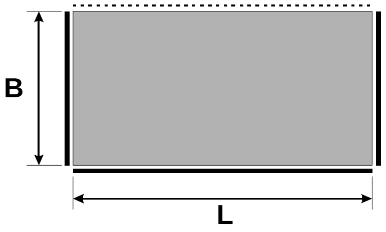
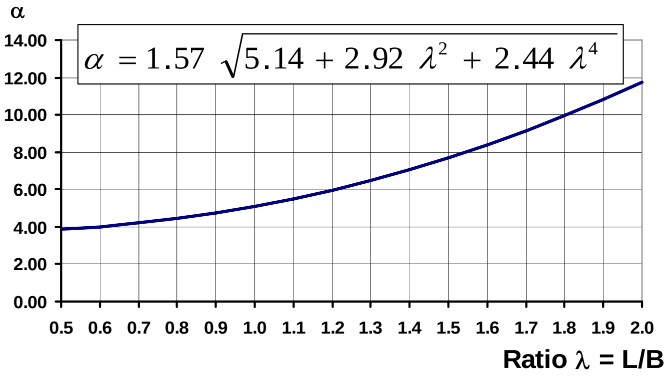
Figure 11: OS-RMS$_{90}$ for 9% Damping

A. Calculation of natural frequency and modal mass of floors and other structures

A.1. Natural Frequency and Modal Mass for Isotropic Plates

The following table gives hand formulas for the determination of the first natural frequency (according to [2]) and the modal mass of plates for different supporting conditions.

For the application of the given equations it is assumed that no lateral deflection at any edges of the plate occurs.

Supporting Conditions: ▬ **clamped** ⋯ **hinged**	**Frequency ; Modal Mass** $f = \frac{\alpha}{L^2}\sqrt{\frac{E\,t^3}{12\cdot\mu\,(1-\upsilon^2)}}$; $M_{\text{mod}} = \beta\cdot M_{tot}$
	 $\alpha = 1.57\cdot(1+\lambda^2)$ β ≈ 0,25 for all λ
	 $\alpha = 1.57\sqrt{1+2.5\,\lambda^2+5.14\,\lambda^4}$ β ≈ 0,20 for all λ
	 $\alpha = 1.57\sqrt{5.14+2.92\,\lambda^2+2.44\,\lambda^4}$

<table>
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<th>Supporting Conditions:

▬ clamped ▭ hinged</th>
<th>Frequency ; Modal Mass
$f = \frac{\alpha}{L^2}\sqrt{\frac{E\,t^3}{12\cdot\mu\,(1-\upsilon^2)}}$; $M_{\text{mod}} = \beta \cdot M_{tot}$</th>
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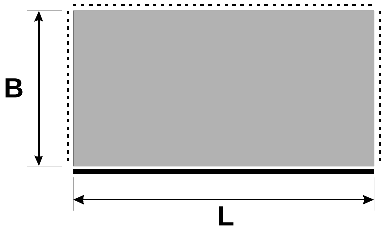

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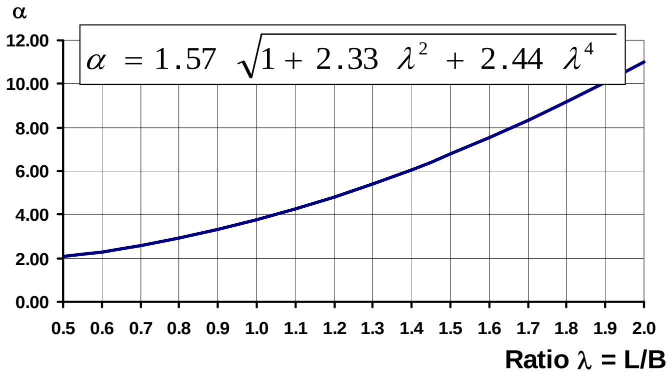

β ≈ 0,22 for all λ
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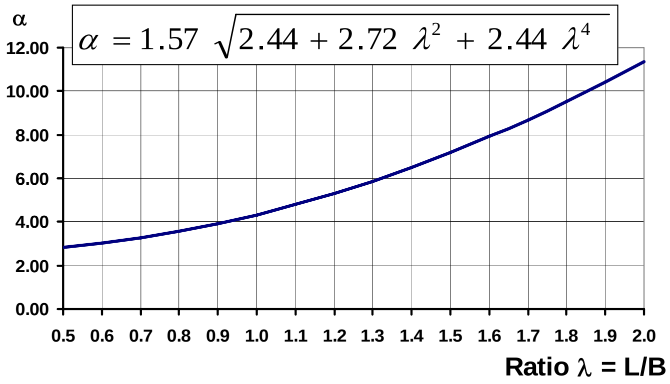

β ≈ 0,21 for all λ
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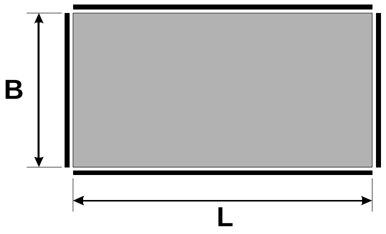

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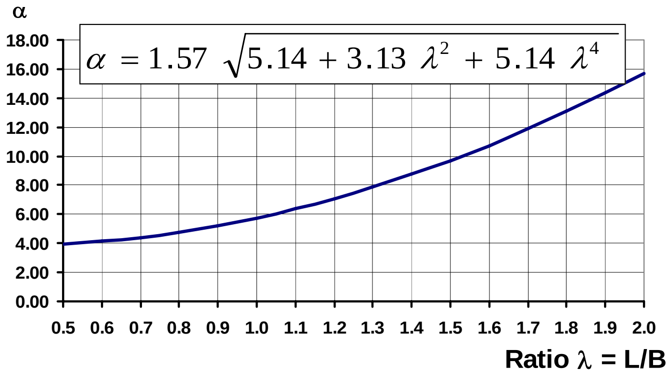

β ≈ 0,17 for all λ
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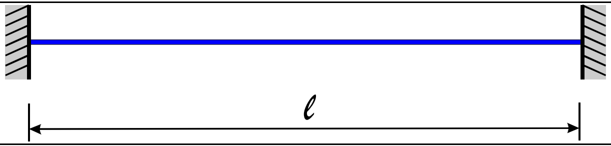
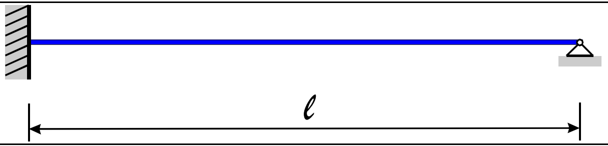
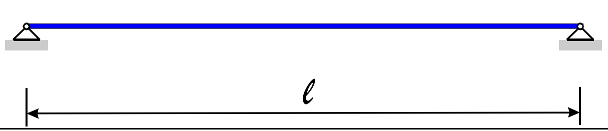
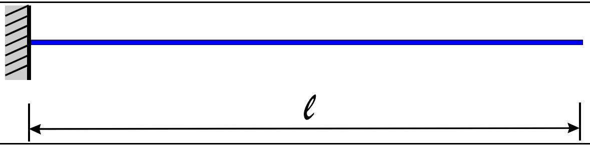
<td>
E Youngs Modulus in N/m²
t Thickness of Plate in m
μ mass of floor including finishing and furniture in kg/m²
ν Poisson ratio
M_{tot} Total mass of floor including finishes and representative variable loading in kg
</td>
</tr>
</table>

A.2. Natural Frequency and Modal Mass for Beams

The first Eigenfrequency of a beam can be determined with the formula according to the supporting conditions from Table 4 with:

E Youngs-Modulus [N/m²]
I Moment of inertia [m^4]
μ distributed mass [kg/m]
ℓ length of beam

Table 4: Determination of the first Eigenfrequency of Beams

Supporting Conditions	Natural Frequency	Modal Mass
	$f = \frac{4}{\pi}\sqrt{\frac{3EI}{0.37\mu l^4}}$	$M_{\text{mod}} = 0{,}41\ \mu\ l$
	$f = \frac{2}{\pi}\sqrt{\frac{3EI}{0.2\mu l^4}}$	$M_{\text{mod}} = 0{,}45\ \mu\ l$
	$f = \frac{2}{\pi}\sqrt{\frac{3EI}{0.49\ \mu l^4}}$	$M_{\text{mod}} = 0{,}5\ \mu\ l$
	$f = \frac{1}{2\pi}\sqrt{\frac{3EI}{0.24\mu l^4}}$	$M_{\text{mod}} = 0{,}64\ \mu\ l$

A.3. Natural Frequency and Modal Mass for Orthotropic Plates

Orthotropic floors as e.g. composite floors with beam in longitudinal direction and a concrete plate in transversal direction have a different stiffness in length and width ($EI_y > EI_x$). An example is given in Figure A.1.

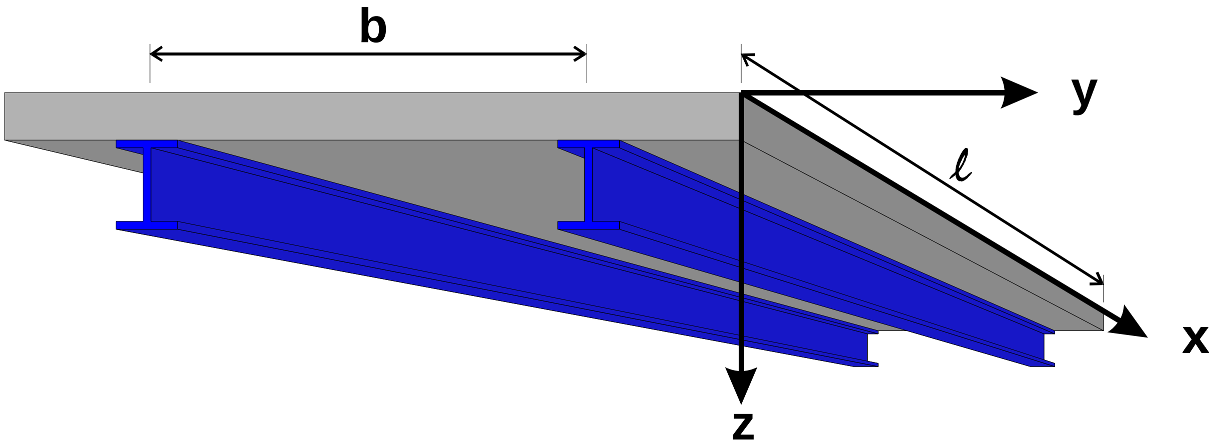


Figure A.1: Dimensions and axis of an orthotropic plate

The first natural frequency of the orthotropic plate being simply supported at all four edges can be determined with:

$$f_1 = \frac{\pi}{2}\sqrt{\frac{EI_y}{\mu\, l^4}}\sqrt{1+\left[2\left(\frac{b}{l}\right)^2+\left(\frac{b}{l}\right)^4\right]\frac{EI_x}{EI_y}}$$

Where:

μ	is the mass per m² in kg/m²,
ℓ	is the length of the floor in m (in x-direction),
b	is the width of the floor in m (in y-direction),
E	is the Youngs-Modulus in N/m²,
I_x	is the moment of inertia for bending about the x-axis in m^4,
Iy	is the moment of inertia for bending about the y-axis in m4.

A.4. Self weight Approach for natural Frequency

The self weight approach is a very practical approximation in cases where the maximum deflection δ_{max} due to self weight loading is already determined, e.g. by finite element calculation.

This method has it origin in the general frequency equation: $f = \frac{1}{2\pi}\sqrt{\frac{K}{M}}$

The stiffness K can be approximated by the assumption:

$$K = \frac{M \cdot g}{\frac{3}{4}\delta_{max}},$$

where

M is the total mass of the vibrating system,

$g = 9.81 \frac{m}{s^2}$ is gravity and

$\frac{3}{4}\delta_{max}$ is the average deflection.

The approximated natural frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{M}} = \frac{1}{2\pi}\sqrt{\frac{4 \cdot g}{3 \cdot \delta_{\max}}} = \frac{18}{\sqrt{\delta_{\max}[mm]}}$$

A.5. Dunkerley Approach for natural Frequency

The Dunkerley approach is an approximation for hand calculations. It is applied when the expected mode shape is complex but can be subdivided into different single modes of which the natural frequency can be determined, see A.1, A.3 and A.2.

Figure 12 shows an example of a composite floor with two simple supported beams and no support at the edges of the concrete plate.

The excepted mode shape is divided into two independent single mode shapes. Both mode shapes have their own natural frequency (f_1 for the vibration of the concrete slab and f_2 for the composite beam).

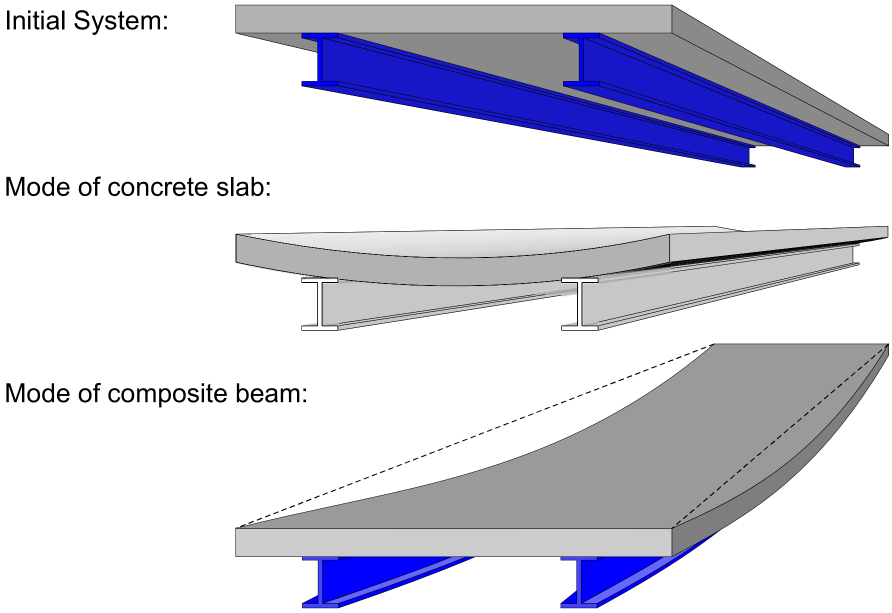


Figure 12: Example for mode shape decomposition

According to Dunkerley the resulting natural frequency f of the total system is:

$$\frac{1}{f^2} = \frac{1}{f_1^2} + \frac{1}{f_2^2} + \frac{1}{f_3^2} + \ldots$$

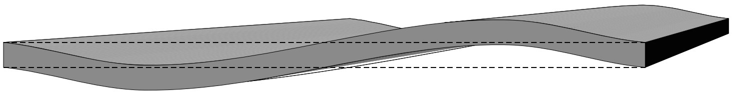
A.6. Approximation of modal mass

The modal mass may be interpreted as the fraction of the total mass of a floor that is activated when the floor oscillates in a specific mode shape. Each mode shape has its specific natural frequency and modal mass.

For the determination of the modal mass the mode shape has to be determined and to be normalized to the maximum deflection. As the mode shape cannot be determined by hand calculations approximations for the first mode are commonly used.

As an alternative to hand calculations Finite Element Analysis is commonly used. If the Finite Element Program does not give modal mass as result of modal analysis the mode shape may be approximated by the application of loads driving the plate into the expected mode shape, see Figure 13.

Expected mode shape:



Application of loads:

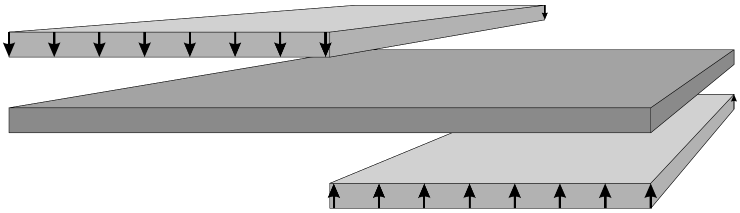


Figure 13: Application of load to obtain approximated load shape (example)

If the mode shape of a floor can be approximated by a normalised function δ(x,y) (i.e. $|\delta(x,y)|_{max.} = 1{,}0$) the corresponding modal mass of the floor can be calculated by the following equation:

$$M_{mod} = \mu \int_F \delta^2(x,y)\, dF$$

Where

μ is the Distribution of mass

$\delta(x,y)$ is the vertical deflection at location x,y

When mode shape deflections are determined by FEA:

$$M_{mod} = \sum_{Nodes\,i} \delta_i^2 \times dM_i$$

Where

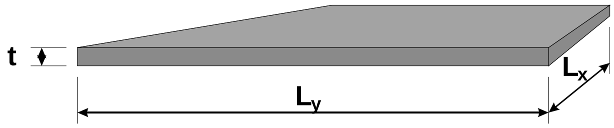
f_i is the vertical deflection at node i (normalised to the maximum deflection)

dM_i is the mass of the floor represented at node i

If the function δ(x,y) represents the exact solution for the mode shape the above described equation also yields to the exact modal mass.

The following gives examples for the determination of modal mass by hand calculation:

Example 1: Plate simply supported at all four edges, $L_x \sim L_y$



- Approximation of the first mode shape:

$$\delta(x,y) = \sin\left(\frac{\pi \times x}{l_x}\right) \times \sin\left(\frac{\pi \times y}{l_y}\right), \quad |\delta(x,y)|_{max.} = 1,0$$

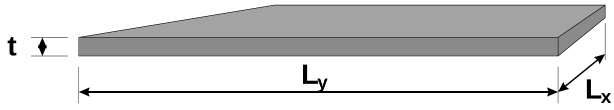
- Mass distribution

$$\mu = \frac{M_{total}}{l_x \times l_y}$$

- Modal mass

$$M_{mod} = \mu \times \int_F \delta^2(x,y) \times dF = \frac{M_{total}}{l_x \times l_y} \times \int_{l_y}\int_{l_x} \sin^2\left(\frac{\pi \times x}{l_x}\right) \times \sin^2\left(\frac{\pi \times y}{l_y}\right) \times dx \times dy = \frac{M_{total}}{4}$$

Example 2: Plate simply supported at all four edges, $L_x << L_y$



- Approximation of the first mode shape:

1. $0 \le y \le \frac{l_x}{2}$ and $l_y - \frac{l_x}{2} \le y \le l_y$:

$$f(x,y) = \sin\left(\frac{\pi \times x}{l_x}\right) \times \sin\left(\frac{\pi \times y}{l_y}\right), \quad |f(x,y)|_{max.} = 1,0$$

2. $\frac{l_x}{2} \le y \le l_y - \frac{l_x}{2}$:

$$\delta(x,y) = \sin\left(\frac{\pi \times x}{l_x}\right) \times 1.0, \quad |\delta(x,y)|_{max.} = 1,0$$

- Mass distribution

$$\mu = \frac{M_{total}}{l_x \times l_y}$$

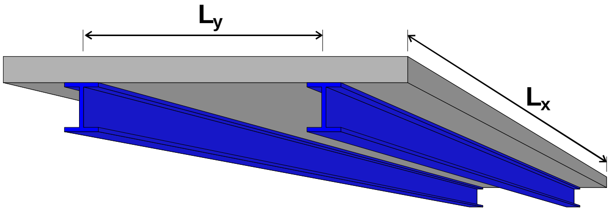
- Modal mass

$$M_{mod} = \mu\times \int_F \delta^2(x,y)\times dF$$

$$= \frac{M_{total}}{l_x\times l_y}\times\left[2\times\int_0^{l_x}\int_0^{y=l_2}\sin^2\left(\frac{\pi\times x}{l_x}\right)\times\sin^2\left(\frac{\pi\times y}{l_y}\right)\times dx\times dy+\int_0^{l_x}\int_0^{l_2-2\frac{l_x}{2}}\sin^2\left(\frac{\pi\times x}{l_x}\right)\times dx\times dy\right]$$

$$= \frac{M_{total}}{4}\times\left(2-\frac{l_x}{l_y}\right)$$

Example 3: Plate spanning in one direction between beams, plate and beams simply supported



- Approximation of the first mode shape:

$$\delta(x,y)=\frac{\delta_x}{\delta}\times\sin\left(\frac{\pi\times x}{l_x}\right)+\frac{\delta_y}{\delta}\times\sin\left(\frac{\pi\times y}{l_y}\right),\ \left|\delta(x,y)\right|_{max.}=1{,}0$$

With

δ_x = Deflection of the beam

δ_y = Deflection of the slab assuming the deflection of the supports (i.e. the deflection of the beam) is zero

$\delta = \delta_x + \delta_y$

- Mass distribution

$$\mu = \frac{M_{total}}{l_x\times l_y}$$

- Modal mass

$$M_{mod} = \mu\times\int_F \delta^2(x,y)\times dF = \frac{M_{total}}{l_x\times l_y}\times\int_{l_x}\int_{l_y}\left[\frac{\delta_x}{\delta}\times\sin\left(\frac{\pi\times x}{l_x}\right)+\frac{\delta_y}{\delta}\times\sin\left(\frac{\pi\times y}{l_y}\right)\right]^2\times dx\times dy$$

$$= M_{total}\times\left[\frac{\delta_x^{\ 2}+\delta_x^{\ 2}}{2\delta}+\frac{8}{\pi^2}\times\frac{\delta_x\times\delta_y}{\delta^2}\right]$$

B. Examples

B.1. Filigree slab with ACB-composite beams (office building)

B.1.1. Description of the Floor

In the first worked example a filigree slab with false-floor in an open plan office is checked for footfall induced vibrations.

Figure 14: Structure building

It is spanning one way over 4.2 m between main beams. Its overall thickness is 160 mm. The main beams are Arcelor Cellular Beams (ACB) which act as composite beams. They are attached to the vertical columns by a full moment connection. The floor plan shown in Figure 15. In Figure 15 the part of the floor which will be considered for the vibration analysis is indicated by the hatched area.

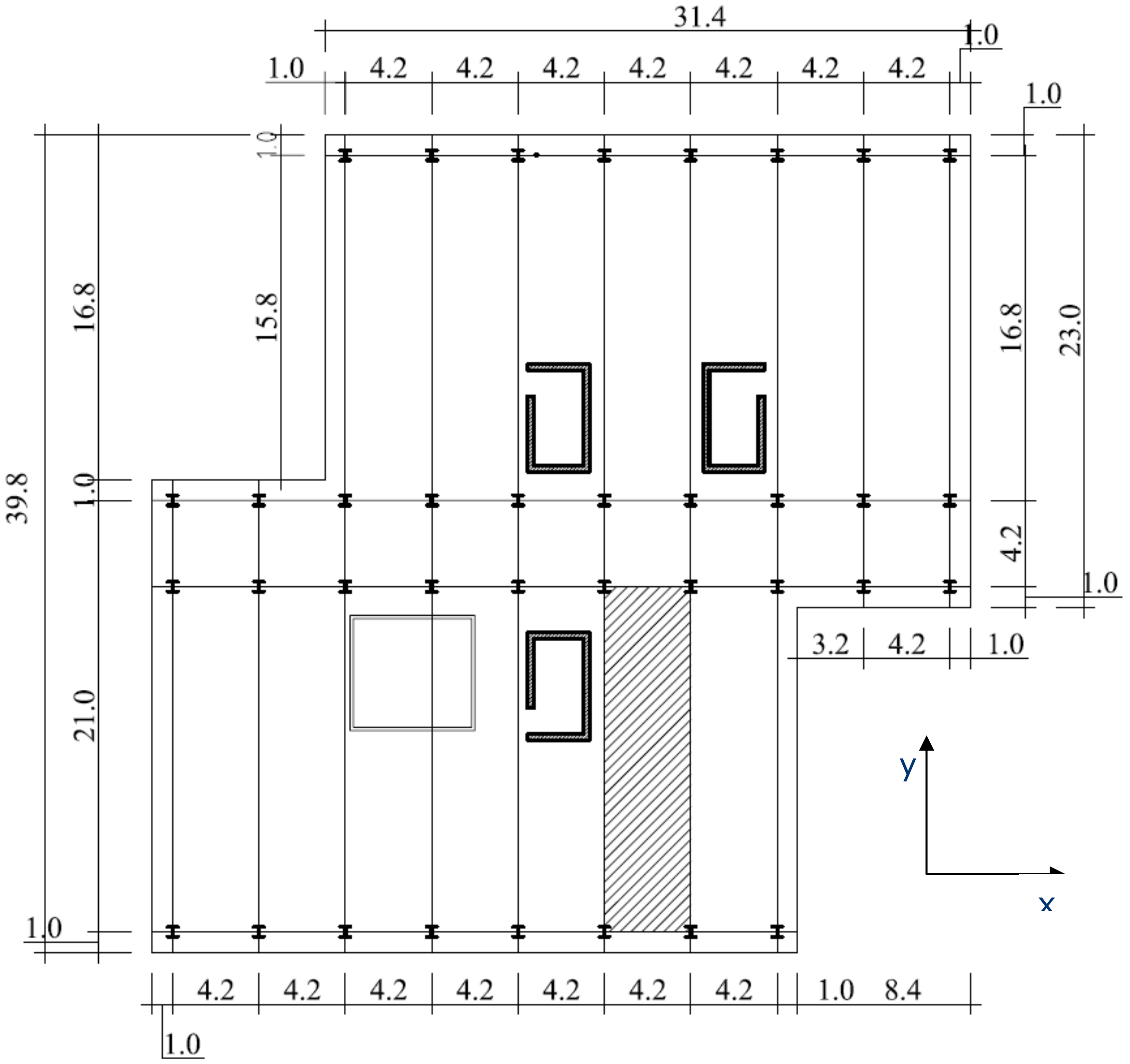


Figure 15: Floor plan

For the main beams with a span of 16.8 m ACB/HEM400 profiles in material S460 have been used, the main beams with the shorter span of 4.2 m are ACB/HEM360 in S460.

The cross beams which are spanning in global x-direction may be neglected for the further calculations, as they do not contribute to the load transfer of the structure.

The nominal material properties are

- Steel S460: E_s = 210 000 N/mm², f_y = 460 N/mm²
- Concrete C25/30: E_{cm} = 31 000 N/mm², f_{ck} = 25 N/mm²

As stated in chapter 0 the nominal Elastic modulus of the concrete will be increased for the dynamic calculations:

$$E_{c,dyn} = 1.1 \times E_{cm} = 34100 \ N / mm^2$$

The expected mode shape of the considered part of the floor which corresponds to the first eigenfrequency is shown in Figure 17. From the mode shape it can be concluded that each field of the concrete slab may be assumed to be simply supported for the further dynamic calculations. Regarding the boundary conditions of the main beams (see beam to column connection, Figure 16) it is assumed that for small amplitudes as they occur in vibration analysis the beam-column connection provides sufficient rotational restrained, i.e. the main beams are considered to be fully fixed.

Figure 16: Beam to column connection

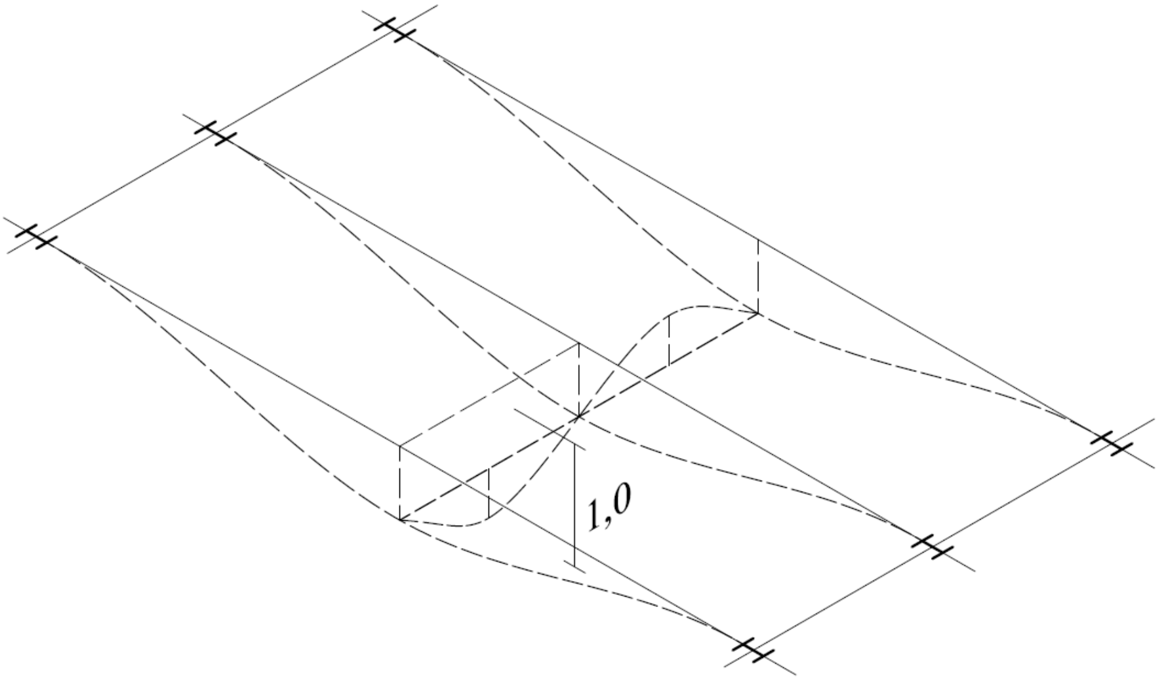


Figure 17: Expected mode shape of the considered part of the floor corresponding to the first eigenfrequency

Section properties

- Slab:
  The relevant section properties of the slab in global x-direction are:

  $$A_{c,x} = 160 \frac{mm^2}{mm}$$

  $$I_{c,x} = 3.41 \times 10^5 \frac{mm^4}{mm}$$

- Main beam:
  Assuming the above described first vibration mode the effective width of the composite beam may be obtained from the following equation:

  $$b_{eff} = b_{eff,1} + b_{eff,2} = \frac{l_0}{8} + \frac{l_0}{8} = 2 \times \frac{0.7 \times 16.8}{8} = 2.94\,m$$

  The relevant section properties of the main beam for serviceability limit state (no cracking) are:

  $$A_{a,netto} = 21936\,mm^2$$

  $$A_{a\,brutto} = 29214\,mm^2$$

  $$A_i = 98320\,mm^2$$

  $$I_i = 5.149 \times 10^9\,mm^4$$

Loads

- Slab:
    - Self weight (includes 1.0 kN/m² for false floor):

      $$g_{slab} = 160 \times 10^{-3} \times 25 + 1.0 = 5 \frac{kN}{m^2}$$

    - Live load: Usually a characteristic live load of 3 kN/m² is recommended for floors in office buildings. The considered fraction of the live load for the dynamic calculation is assumed to be approx. 10% of the full live load, i.e. for the vibration check it is assumed that

      $$q_{slab} = 0.1 \times 3.0 = 0.3 \frac{kN}{m^2}$$

- Main beam:
    - Self weight (includes 2.00 kN/m for ACB):

      $$g_{beam} = 5.0 \times \frac{4.2}{2} \times 2 + 2.0 = 23.00 \frac{kN}{m^2}$$

    - Live load:

      $$q_{slab} = 0.3 \times \frac{4.2}{2} \times 2 = 0.63 \frac{kN}{m^2}$$

B.1.2. Determination of dynamic floor characteristics

Eigenfrequency

The first eigenfrequency is calculated based on the self weight approach. The maximum total deflection may be obtained by superposition of the deflection of the slab and the deflection of the main beam, i.e.

$$\delta_{total} = \delta_{slab} + \delta_{beam}$$

With

$$\delta_{slab} = \frac{5 \times (5.0 + 0.3) \times 10^{-3} \times 4200^4}{384 \times 34100 \times 3.41 \times 10^5} = 1.9\,mm$$

$$\delta_{beam} = \frac{1 \times (23.0 + 0.63) \times 16800^4}{384 \times 210000 \times 5.149 \times 10^9} = 4.5mm$$

the total deflection is

$$\delta_{total} = 1.9 + 4.5 = 6.4\,mm$$

Thus the first eigenfrequency may be obtained from

$$f_1 = \frac{18}{\sqrt{6.4}} = 7.1\,Hz$$

Modal Mass

The total mass of the slab is

$$M_{total} = (5 + 0.3) \times 10^2 \times 16.8 \times 4.2 = 37397\,kg$$

According to chapter A.6, example 3 the modal mass of the considered slab may be calculated as

$$M_{\text{mod}} = 37397 \times \left[\frac{1.9^2 + 4.5^2}{2 \times 6.4^2} + \frac{8}{\pi^2} \times \frac{1.9 \times 4.5}{6.4^2} \right] = 17220kg$$

Damping

The damping ratio of the steel-concrete slab with false floor is determined according to table 1:

$$D = D_1 + D_2 + D_3 = +1 + 1 + 1 = 3\,\%$$

With

$D_1 = 1{,}0$ (steel-concrete slab)
$D_2 = 1{,}0$ (open plan office)
$D_3 = 1{,}0$ (false floor)

B.1.3. Assessment

Based on the above calculated modal properties the floor is classified as class C (Figure 5). The expected OS-RMS value is approx. 0.5 mm/s.

According to Table 1 class C is classified as being suitable for office buildings, i.e. the requirements are fulfilled.

B.2. Three storey office building

B.2.1. Description of the Floor

The floor of this office building, Figure 18, span 15m from edge beam to edge beam. In the regular area these secondary floor beams have IPE600 sections are laying in a distance of 2.5m. Primary edge beam which span 7.5m from column to column have also IPE600 section, see Figure 19.

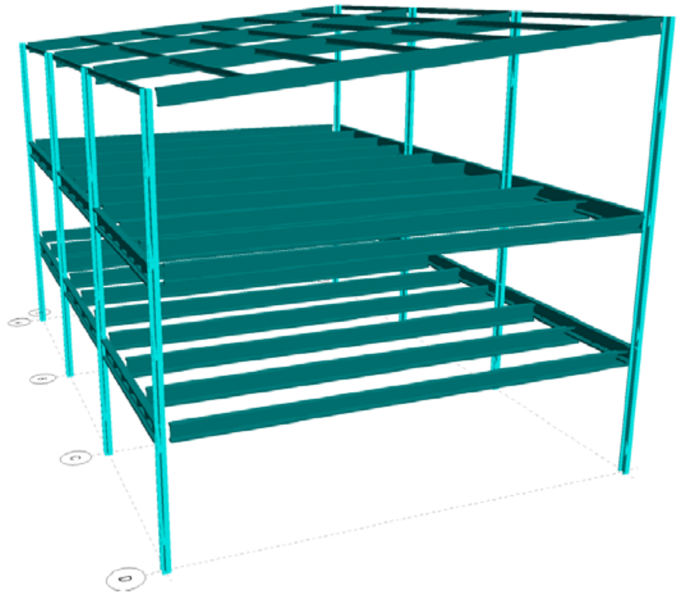


Figure 18: Building overview

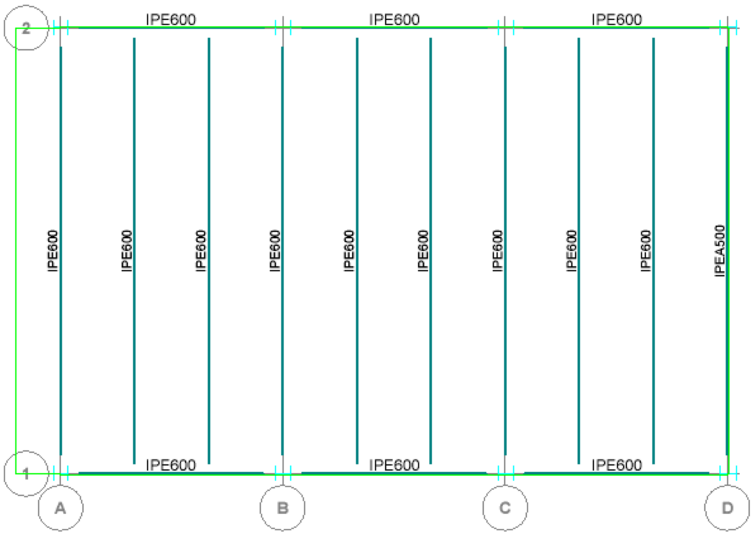


Figure 19: Steel section of the floor

The plate of the floor is a composite plate of 15cm total thickness with steel sheets COFRASTRA 70, Figure 20.

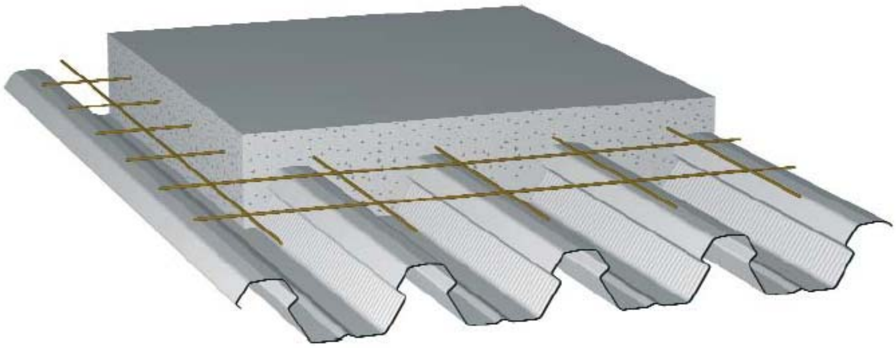


Figure 20: Floor set up

The nominal material properties are

- Steel S235: E_s = 210 000 N/mm², f_y = 235 N/mm²
- Concrete C25/30: E_{cm} = 31 000 N/mm², f_{ck} = 25 N/mm²

$$E_{c,dyn} = 1.1 \times E_{cm} = 34100\ N/mm^2$$

Section properties

- Slab (transversal to beam):
  A = 1170 cm²/m
  I = 20 355 cm^4/m
  g = 3.5 kN/m²
  Δg = 0.5 kN/m²

- Composite beam (b_{eff} = 2,5m; E=210000 N/mm²):
  A = 468 cm²
  I = 270 089 cm^4
  g = (3.5+0.5) x 2.5 + 1.22 = 11.22 kN/m²

Loads

- Slab (transversal to beam):
  g + Δg = 4.0 kN/m² (permanent load)
  q = 3.0 x 0.1 = 0.3 kN/m² (10% of full live load)
  p_{total} = 4.3 kN/m²

- Composite beam (b_{eff} = 2,5m; E=210000 N/mm²):
  g = 11.22 kN/m²
  q = 0.3 x 2.5 = 0.75 kN/m
  p_{total} = 11.97 kN/m

B.2.2. Determination of dynamic floor characteristics

Supporting conditions

The secondary beam are ending in the primary beams which are open sections with low torsional stiffness. Thus these beams may be assumed to be simple supported.

Eigenfrequency

For this example the supporting conditions are determined on two ways. The first method is the application of the beam formula neglecting the transversal stiffness of the floor.

The second method is the self weight method considering the transversal stiffness.

- Application of the beam equation (Chapter A.2):

$$p = 11.97\ [kN/m] \Rightarrow \mu = 11.97 \times 1000\ [kg\ m/s^2/m]/9.81[m/s^2] = 1220\ [kg/m]$$

$$f = \frac{2}{\pi}\sqrt{\frac{3EI}{0.49\ \mu l^4}} = \frac{2}{\pi}\sqrt{\frac{3\times 210000\times 10^6\ [N/m^2]\times 270089\times 10^{-8}[m^4]}{0.49\times 1220\ [kg/m]\times 15^4\ [m^4]}} = 4{,}77\ Hz$$

- Application of the equation for orthotropic plates (Chapter A.3):

$$f_1 = \frac{\pi}{2}\sqrt{\frac{EI_y}{m\, l^4}}\sqrt{1+\left[2\left(\frac{b}{l}\right)^2+\left(\frac{b}{l}\right)^4\right]\frac{EI_x}{EI_y}}$$

$$= \frac{\pi}{2}\sqrt{\frac{210000\times 10^6\times 270089\times 10^{-8}}{1220\times 15^4}}\sqrt{1+\left[2\left(\frac{2.5}{15}\right)^2+\left(\frac{2.5}{15}\right)^4\right]\frac{3410\times 20355}{21000\times 270089}}$$

$$= 4.76\times 1.00 = 4{,}76$$

- Application of the self weight approach (Chapter A.4):

$$\delta_{total} = \delta_{slab} + \delta_{beam}$$

$$\delta_{slab} = \frac{5\times 4.3\times 10^{-3}\times 2500^4}{384\times 34100\times 2.0355\times 10^5} = 0.3\, mm$$

$$\delta_{beam} = \frac{5\times 11.97\times 15000^4}{384\times 210000\times 270089\times 10^4} = 13.9 mm$$

$$\delta_{total} = 0.3 + 13.9 = 14.2\, mm$$

$$\Rightarrow \qquad f_1 = \frac{18}{\sqrt{14.2}} = 4.78\, Hz$$

Modal Mass

The determination of the eigenfrequency above showed that the load bearing behaviour of the floor can be approximated by a simple beam model. Thus this model is taken for the determination of the modal mass:

$$M_{\text{mod}} = 0{,}5\ \mu\ l = 0{,}5 \times 1220 \times 15 = 9150\ kg$$

Damping

The damping ratio of the steel-concrete slab with false floor is determined according to table 1:

$$D = D_1 + D_2 + D_3 = +1 + 1 + 1 = 3\,\%$$

With

$D_1 = 1{,}0$ (steel-concrete slab)
$D_2 = 1{,}0$ (open plan office)
$D_3 = 1{,}0$ (ceiling under floor)

B.2.3. Assessment

Based on the above calculated modal properties the floor is classified as class D (Figure 5). The expected OS-RMS value is approx. 3.2 mm/s.

According to Table 1 class D is classified as being suitable for office buildings, i.e. the requirements are fulfilled.